

RESEARCH ARTICLE**Explicit Constructions of Hetro Cordial Labelings for Selected Graphs**S. Jackson ^{a,*}, S. Rohini ^b, M. Abirami ^c^aAssistant Professor, P. G and Research Department of Mathematics, V. O. Chidambaram College, Thoothukudi-628008, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, India.^bM. Sc in Mathematics (2024-26), P.G and Research Department of Mathematics, V. O. Chidambaram College, Thoothukudi-628008, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, India.^cPh. D Scholar of Mathematics, P.G and Research Department of Mathematics, V. O. Chidambaram College, Thoothukudi-628008, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, India.**Abstract**

Hetro Cordial Labeling (HCL) is a graph labeling technique in which a bijection assigns labels from 0, 1 to the vertices of a graph, and the induced edge labeling is determined by the labels of the corresponding end vertices. A graph is said to admit HCL when the numbers of vertices and edges receiving the labels 0 and 1 differ by at most one. In this paper, the Hetro Cordial Labeling property is investigated for several classes of graphs. Explicit vertex and induced edge labeling schemes are constructed for the Lotus inside a circle (LC_n), Twig (T_{g_n}), Krackhardt Kite, Bull, and Fish graphs. For each graph, the proposed labeling satisfies the required balance conditions on the vertex and edge labels. These results establish that the considered graphs admit Hetro Cordial Labeling and contribute to the study of labeling properties of specific graph structure.

Keywords: Hetro Cordial Labeling, Graph structure, Hetro Cordial Labeling Graph.

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This is an open-access article distributed under the terms of the [Creative Commons Attribution–NonCommercial 4.0 International License \(CC BY-NC 4.0\)](#).**1. Introduction**

Graph labeling is an important area of graph theory in which labels are assigned to the vertices, edges, or both, according to prescribed conditions [2]. Among the various graph labeling techniques, cordial labeling, introduced by Cahit, is a binary labeling scheme in which the numbers of vertices and edges receiving the two possible labels differ by at most one [1]. The concept of cordial labeling has motivated the development of several related labeling techniques and has become an active topic in graph theory. In particular, Hetro Cordial Labeling (HCL) was discussed to investigate labeling properties based on the balanced distribution of labels on the vertices and edges of a graph

[3]. Further studies investigated HCL for different families of graphs, including tree-related and path-related graphs [4], [5].

Recent research has continued to extend the theory of cordial labelings through different labeling conditions and graph structures. Idrees et al. investigated certain prime cordial families of graphs, demonstrating the continued development of cordial labeling techniques and their applications to specific graph families [6]. Ullah et al. studied 3-total edge product cordial labeling for the stellation of square grid graphs, further illustrating the development of cordial-type labeling schemes involving both vertices and edges [7]. These studies show that cordial labeling remains an active research area in which new labeling conditions are formulated to investigate the structural properties of various classes of graphs [13].

Further developments have considered alternative forms

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of cordial labeling. Elrokhi et al. studied cordial and total cordial labeling for corona products of paths and second-order lemniscate graphs [8]. Subsequently, Elrokhi et al. introduced a problem concerning permuted cordial labeling of graphs using permutation groups together with the concept of cordiality [9]. More recently, Abdel-Aal and Bashammakh investigated equivalent cordial labeling graphs and established relationships between cordial, signed product cordial, and logical cordial labeling [10]. These investigations demonstrate the continuing expansion of cordial labeling into several related and structurally distinct labeling frameworks.

Motivated by these developments, the present paper focuses on Hetro Cordial Labeling of selected graphs. In particular, we investigate the Hetro Cordial Labeling property for the Lotus inside a circle graph, Twig graph, Krackhardt Kite graph, Bull graph, and Fish graph. For each graph, an appropriate vertex labeling is constructed, and the corresponding induced edge labeling is examined to verify the required cordiality conditions. The results provide further examples of graphs admitting the HCL and contribute to the continuing study of cordial-type labeling properties of graph families. Recent work on topological and other variants of cordial labeling further indicates the broadening scope of this area [12].

2. Preliminaries

Let $G = (V(G), E(G))$ be a finite, simple, undirected graph [1]. For a binary vertex labeling $f : V(G) \rightarrow \{0, 1\}$, let $v_f(i)$ denote the number of vertices labeled i , and let $e_f(i)$ denote the number of edges receiving the induced label i , where $i \in \{0, 1\}$. These quantities are used throughout the proofs to verify the required balance conditions.

A binary vertex labeling $f : V(G) \rightarrow \{0, 1\}$ is called a **cordial labeling** if each edge $uv \in E(G)$ receives the induced label defined in Equation (1) and the vertex and edge labels satisfy the balance conditions in Equation (2).

$$f^*(uv) = |f(u) - f(v)|, \quad uv \in E(G). \quad (1)$$

$$|v_f(0) - v_f(1)| \leq 1, \quad |e_f(0) - e_f(1)| \leq 1. \quad (2)$$

Thus, adjacent vertices with identical labels induce edge label 0, while vertices with different labels induce edge label 1.

[3] Let $G = (V, E)$ be a graph. A function $f : V(G) \rightarrow \{0, 1\}$ is called a Hetro Cordial Labeling of G if its induced edge labeling is given by Equation (3) and both counting conditions in Equation (4) hold.

$$f^*(uv) = |f(u) - f(v)|, \quad uv \in E(G). \quad (3)$$

$$|v_f(0) - v_f(1)| \leq 1, \quad |e_f(0) - e_f(1)| \leq 1. \quad (4)$$

A graph admitting such a labeling is referred to as a Hetro Cordial Graph (HCG). Equations (3) and (4) provide the verification criteria applied to each graph in Section 3.

The lotus inside a circle LC_n is obtained from a cycle C_n with vertices $u_1, \dots, u_n$ and a star $K_{1,n}$ with center w and leaves $v_1, \dots, v_n$, by joining each leaf v_i to u_i and u_{i+1} , with indices taken modulo n . Its vertex and edge sets are specified in Equations (5) and (6) below.

A twig T_{g_n} is obtained from a path on n vertices by attaching two pendant vertices to each internal path vertex. Accordingly, T_{g_n} has n path vertices and $2(n-2)$ pendant vertices.

The Krackhardt kite graph is a graph with ten vertices and eighteen edges. The specific vertex and edge representation considered here is given in Equation (18).

The bull graph is a planar, undirected graph having five vertices and five edges. It comprises a triangle with two disjoint pendant edges.

The **Fish graph** considered in this study contains six vertices and seven edges, with its edge set given in Equation (26).

3. Result Discussion

The following theorems provide explicit binary vertex labelings and verify the corresponding induced edge-label distributions using Equations (3) and (4). In each proof, the counting expressions establish the balance of both label classes.

Theorem 3.1. *The lotus inside a circle graph LC_n admits HCL for $n \geq 3$.*

Proof. Let $G = LC_n$, where the vertex set is defined by Equation (5):

$$V(LC_n) = \{u_z, v_z : 1 \leq z \leq n\} \cup \{w\}. \quad (5)$$

Using cyclic subscripts ($u_{n+1} = u_1$ and $v_{n+1} = v_1$), the edge set is expressed in Equation (6):

$$\begin{aligned} E(LC_n) = & \{u_z u_{z+1} : 1 \leq z \leq n\} \\ & \cup \{u_z v_z, u_z v_{z+1} : 1 \leq z \leq n\} \\ & \cup \{w v_z : 1 \leq z \leq n\}. \end{aligned} \quad (6)$$

Define a binary vertex labeling $\psi : V(G) \rightarrow \{0, 1\}$ as given in Equation (7):

$$\psi(u_z) = 0, \quad \psi(v_z) = 1 \quad (1 \leq z \leq n), \quad \psi(w) = 1. \quad (7)$$

Applying the induced labeling rule in Equation (3), the edge labels are determined by Equation (8):

$$\begin{aligned} \psi^*(u_z u_{z+1}) &= 0, & \psi^*(u_z v_z) &= 1, \\ \psi^*(u_z v_{z+1}) &= 1, & \psi^*(w v_z) &= 0, \end{aligned} \quad 1 \leq z \leq n. \quad (8)$$

The numbers of vertices and edges in the two label classes follow from Equations (7) and (8), respectively, as summarized in Equation (9):

$$\begin{aligned} v_\psi(0) &= n, & v_\psi(1) &= n+1, \\ e_\psi(0) &= 2n, & e_\psi(1) &= 2n. \end{aligned} \quad (9)$$

Consequently, $|v_\psi(0) - v_\psi(1)| = 1$ and $|e_\psi(0) - e_\psi(1)| = 0$. Both conditions of Equation (4) hold; hence LC_n is an HCG.

Example 3.1. Figure 1 illustrates the HCL of LC_4 , depicting the binary vertex labels and the corresponding induced edge labels constructed in the preceding proof.

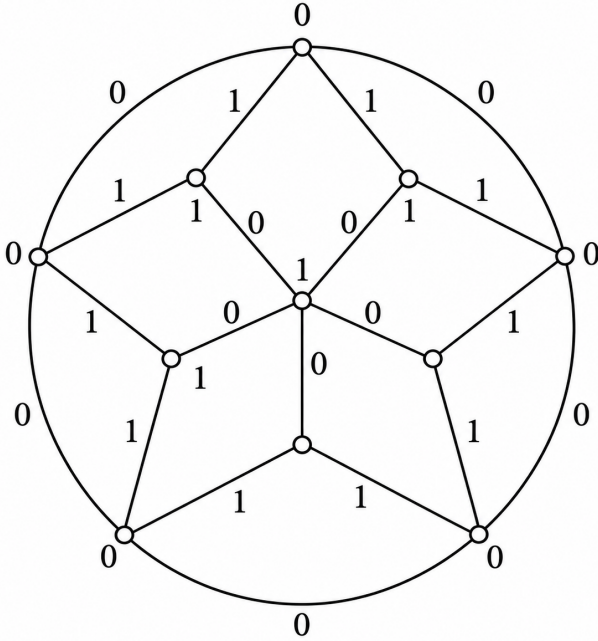


Figure 1: HCL of LC_4

Theorem 3.2. The twig graph T_{g_n} admits an HCL for every odd integer $n \geq 3$.

Proof. Let $G = T_{g_n}$. Its vertex and edge sets are given in Equations (10) and (11), respectively:

$$V(T_{g_n}) = \{u_s : 1 \leq s \leq n\} \cup \{v_s, w_s : 1 \leq s \leq n-2\}. \quad (10)$$

$$E(T_{g_n}) = \{u_s u_{s+1} : 1 \leq s \leq n-1\} \cup \{u_{s+1} v_s, u_{s+1} w_s : 1 \leq s \leq n-2\}. \quad (11)$$

Define $\phi : V(G) \rightarrow \{0, 1\}$. The labels of path vertices follow the modular rule in Equation (12):

$$\phi(u_s) = \begin{cases} 0, & s \equiv 2, 3 \pmod{4}, \\ 1, & s \equiv 0, 1 \pmod{4}, \end{cases} \quad 1 \leq s \leq n. \quad (12)$$

The pendant vertices are assigned complementary labels according to Equation (13):

$$\phi(v_s) = 0, \quad \phi(w_s) = 1, \quad 1 \leq s \leq n-2. \quad (13)$$

For consecutive path vertices, Equation (3) yields the alternating induced labels shown in Equation (14):

$$\phi^*(u_s u_{s+1}) = \begin{cases} 0, & s \equiv 0 \pmod{2}, \\ 1, & s \equiv 1 \pmod{2}, \end{cases} \quad 1 \leq s \leq n-1. \quad (14)$$

The induced labels on the first pendant-edge family are specified in Equation (15):

$$\phi^*(v_s u_{s+1}) = \begin{cases} 0, & s \equiv 1, 2 \pmod{4}, \\ 1, & s \equiv 0, 3 \pmod{4}, \end{cases} \quad 1 \leq s \leq n-2. \quad (15)$$

The second pendant-edge family receives the complementary labels in Equation (16):

$$\phi^*(w_s u_{s+1}) = \begin{cases} 1, & s \equiv 1, 2 \pmod{4}, \\ 0, & s \equiv 0, 3 \pmod{4}, \end{cases} \quad 1 \leq s \leq n-2. \quad (16)$$

Because n is odd, the path has an odd number of vertices and an even number of edges. Equation (12) assigns the two vertex labels to the path with a difference of one; the pendant vertices in Equation (13) contribute equally to both vertex-label classes. Moreover, Equation (14) gives $(n-1)/2$ edges of each label along the path, while Equations (15) and (16) give one edge of each label for every internal path vertex. Thus the label counts satisfy Equation (17):

$$\begin{aligned} |v_\phi(0) - v_\phi(1)| &= 1, \\ e_\phi(0) &= e_\phi(1) = \frac{n-1}{2} + (n-2). \end{aligned} \quad (17)$$

Therefore both inequalities in Equation (4) are satisfied, and T_{g_n} is an HCG.

Example 3.2. The HCL of the twig graph T_{g_5} is illustrated in Figure 2, showing the assigned labels for its path and pendant vertices and the resulting edge labels.

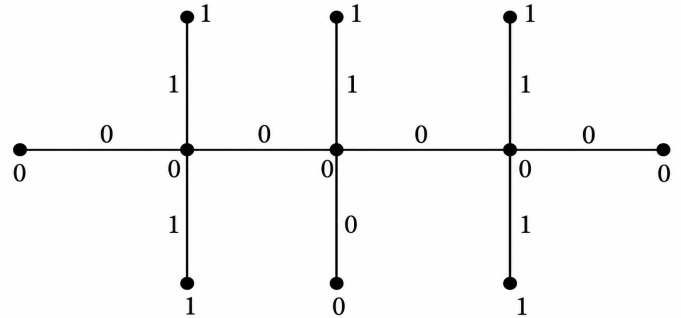


Figure 2: HCL of T_{g_5}

Theorem 3.3. The Krackhardt kite graph with ten vertices and eighteen edges is an HCG.

Proof. Let G be the Krackhardt kite graph with $V(G) = \{u_h : 1 \leq h \leq 10\}$. Its eighteen edges are listed in Equation (18):

$$E(G) = \{u_1 u_2, u_1 u_8, u_5 u_8, u_2 u_3, u_1 u_3, u_6 u_8, u_3 u_4, u_2 u_8, u_9 u_{10}, u_4 u_5, u_3 u_5, u_7 u_8, u_5 u_6, u_3 u_8, u_5 u_7, u_6 u_7, u_4 u_9, u_1 u_7\}. \quad (18)$$

Define the vertex labeling $\eta : V(G) \rightarrow \{0, 1\}$ by Equation (19):

$$\begin{aligned} \eta(u_1) = 0, \quad \eta(u_2) = 1, \quad \eta(u_3) = 0, \quad \eta(u_4) = 0, \\ \eta(u_5) = 0, \quad \eta(u_6) = 1, \quad \eta(u_7) = 0, \quad \eta(u_8) = 1, \\ \eta(u_9) = 1, \quad \eta(u_{10}) = 1. \end{aligned} \quad (19)$$

Applying Equation (3) to the edges in Equation (18) gives the induced labels listed in Equation (20):

$$\begin{aligned} \eta^*(u_1u_2) = 1, \quad \eta^*(u_1u_8) = 1, \quad \eta^*(u_5u_8) = 1, \\ \eta^*(u_2u_3) = 1, \quad \eta^*(u_1u_3) = 0, \quad \eta^*(u_6u_8) = 0, \\ \eta^*(u_3u_4) = 0, \quad \eta^*(u_2u_8) = 0, \quad \eta^*(u_9u_{10}) = 0, \\ \eta^*(u_4u_5) = 0, \quad \eta^*(u_3u_5) = 0, \quad \eta^*(u_7u_8) = 1, \\ \eta^*(u_5u_6) = 1, \quad \eta^*(u_3u_8) = 1, \quad \eta^*(u_5u_7) = 0, \\ \eta^*(u_6u_7) = 1, \quad \eta^*(u_4u_9) = 1, \quad \eta^*(u_1u_7) = 0. \end{aligned} \quad (20)$$

The vertex assignments in Equation (19) produce five vertices of each label, and the eighteen entries in Equation (20) comprise nine edges of each label. Consequently, the counts are as stated in Equation (21):

$$v_\eta(0) = v_\eta(1) = 5, \quad e_\eta(0) = e_\eta(1) = 9. \quad (21)$$

Both differences in Equation (4) are zero. Hence, the Krackhardt kite graph is an HCG.

Example 3.3. Figure 3 presents the HCL of the Krackhardt kite graph, illustrating the vertex assignments and induced edge-labeling construction used in the theorem.

Theorem 3.4. The bull graph with five vertices and five edges is an HCG.

Proof. Let G be the bull graph. Its vertex and edge sets are given by Equation (22):

$$\begin{aligned} V(G) &= \{u_w : 1 \leq w \leq 5\}, \\ E(G) &= \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_2u_4\}. \end{aligned} \quad (22)$$

Define $\psi : V(G) \rightarrow \{0, 1\}$ by the assignments in Equation (23):

$$\begin{aligned} \psi(u_1) = 0, \quad \psi(u_2) = 1, \quad \psi(u_3) = 1, \\ \psi(u_4) = 1, \quad \psi(u_5) = 0. \end{aligned} \quad (23)$$

The corresponding induced edge labeling, computed using Equation (3), is reported in Equation (24):

$$\begin{aligned} \psi^*(u_1u_2) = 1, \quad \psi^*(u_2u_3) = 0, \quad \psi^*(u_3u_4) = 0, \\ \psi^*(u_4u_5) = 1, \quad \psi^*(u_2u_4) = 0. \end{aligned} \quad (24)$$

Counting the labels in Equations (23) and (24) gives Equation (25):

$$v_\psi(0) = 2, \quad v_\psi(1) = 3, \quad e_\psi(0) = 3, \quad e_\psi(1) = 2. \quad (25)$$

Thus $|v_\psi(0) - v_\psi(1)| = 1$ and $|e_\psi(0) - e_\psi(1)| = 1$. By Equation (4), the bull graph is an HCG.

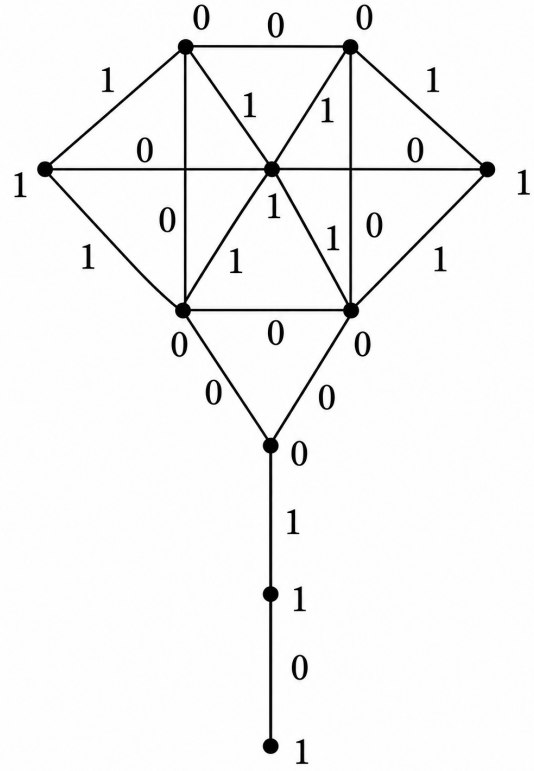


Figure 3: HCL of the Krackhardt kite graph.

Example 3.4. The binary vertex labeling and corresponding induced edge labeling of the bull graph are illustrated in Figure 4.

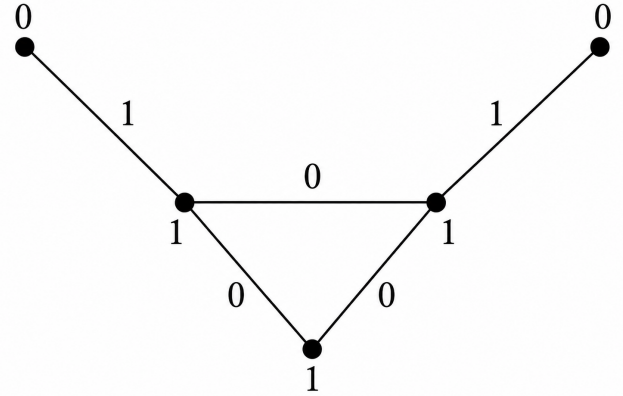


Figure 4: HCL of the bull graph.

Theorem 3.5. The Fish graph with six vertices and seven edges is an HCG.

Proof. Let G be the Fish graph. Its vertex and edge sets are specified in Equation (26):

$$\begin{aligned} V(G) &= \{u_t : 1 \leq t \leq 6\}, \\ E(G) &= \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_1u_3, u_3u_6\}. \end{aligned} \quad (26)$$

Define the binary labeling $\tau : V(G) \rightarrow \{0, 1\}$ as given in Equation (27):

$$\begin{aligned} \tau(u_1) &= 1, & \tau(u_2) &= 0, & \tau(u_3) &= 1, \\ \tau(u_4) &= 1, & \tau(u_5) &= 0, & \tau(u_6) &= 0. \end{aligned} \quad (27)$$

Using Equation (3), the seven induced edge labels are obtained in Equation (28):

$$\begin{aligned} \tau^*(u_1u_2) &= 1, & \tau^*(u_2u_3) &= 1, & \tau^*(u_3u_4) &= 0, \\ \tau^*(u_4u_5) &= 1, & \tau^*(u_5u_6) &= 0, & \tau^*(u_1u_3) &= 0, \\ \tau^*(u_3u_6) &= 1. \end{aligned} \quad (28)$$

The respective vertex- and edge-label totals are presented in Equation (29):

$$v_\tau(0) = 3, \quad v_\tau(1) = 3, \quad e_\tau(0) = 3, \quad e_\tau(1) = 4. \quad (29)$$

Therefore $|v_\tau(0) - v_\tau(1)| = 0$ and $|e_\tau(0) - e_\tau(1)| = 1$. Equation (4) is satisfied, so the Fish graph is an HCG.

Example 3.5. Figure 5 illustrates the HCL of the Fish graph using the vertex and induced edge labels established in the proof.

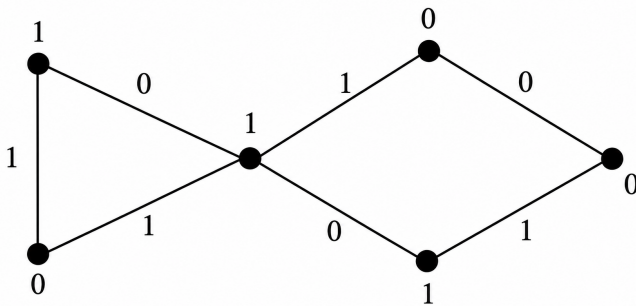


Figure 5: HCL of the fish graph.

4. Conclusion

Our research team has successfully demonstrated the HCL (Hetero-Cordial Labeling) of several important graph classes, including the Lotus inside a Circle graph (LC_n), Twig graph (Tg_n), Bull graph, Fish graph, and Krachardt Kite graph. Considerable effort has been devoted to systematically analyzing the structural properties of each graph and developing effective, innovative, and appropriate labeling strategies. The results obtained through this comprehensive investigation highlight the applicability and effectiveness of HCL across diverse graph structures. These findings further demonstrate our research expertise in graph labeling theory and contribute meaningful insights toward the advancement and development of labeling techniques in graph theory.

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CRediT authorship contribution statement

S. Jackson: Conceptualization, Methodology, Supervision, Validation & Writing- editing. **S. Rohini:** Methodology & Visualization. **M. Abhirami:** Conceptualization, Methodology, Visualization, Writing – original draft.

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