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RESEARCH ARTICLE

Fuzzy Graph Theory and Neuromorphic Graph Models for Uncertain Mathematical Systems

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Abstract

Uncertain mathematical systems frequently involve ambiguous graph relationships, incomplete connectivity structures, nonlinear uncertainty propagation, and dynamically evolving graph interactions that cannot be effectively represented using classical deterministic graph theory. To address these limitations, this research proposed a hybrid Fuzzy-Neuromorphic Graph Framework (FNGF) integrating fuzzy graph theory, adaptive neuromorphic spike propagation, entropy-aware graph optimization, and spectral graph stability analysis within a unified mathematical architecture. The proposed framework incorporates adaptive fuzzy membership modeling, spike-driven graph learning, synaptic graph optimization, and entropy-guided graph sparsification mechanisms for intelligent uncertainty-aware graph computation. Experimental evaluations were conducted using 5000 synthetically generated uncertain graph instances under varying graph perturbation levels and uncertainty densities. The proposed framework achieved uncertainty classification accuracies of 97.82%, 94.12%, and 90.37% under low, medium, and high graph noise environments, respectively. Furthermore, the framework demonstrated 95.84% graph stability prediction accuracy, approximately 60.29% entropy reduction, and nearly 79% reduction in graph computational energy consumption compared to conventional graph neural systems. The entropy-aware graph pruning mechanism significantly improved graph sparsification and graph robustness against topological perturbations, while sparse spike-driven graph propagation substantially enhanced adaptive convergence efficiency. The proposed framework establishes a mathematically rigorous, scalable, interpretable, and energy-efficient intelligent graph architecture suitable for future uncertain intelligent systems, adaptive optimization environments, neuromorphic communication systems, and large-scale uncertainty-aware computational networks.

Keywords: Fuzzy graph theory, Neuromorphic graph systems, Uncertain mathematical systems, Spiking graph neural networks, Entropy-aware graph optimization, Spectral graph analysis, Energy-efficient graph intelligence.

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1. Introduction

Mathematical modeling of complex systems frequently involves uncertainty, ambiguity, incomplete information, nonlinear interactions, and dynamically evolving relationships. Classical graph theory has long served as one of the most important mathematical tools for representing relational structures in communication systems, transportation networks, biological systems, optimization environments, and intelligent computational frameworks [1], [2]. However, traditional crisp graph models assume deterministic vertex and edge relationships, making them inadequate for representing uncertain real-world systems where connectivity and interactions cannot always be expressed using binary structures [5], [23].

To address these limitations, fuzzy graph theory emerged as an extension of classical graph theory through the in-

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Mathematical symbols and notations.

Symbol	Description
G	Hybrid fuzzy-neuromorphic graph structure
V	Set of graph vertices/nodes
E	Set of graph edges
σ_i	Fuzzy membership degree of graph node i
ν_i	Fuzzy non-membership degree of graph node i
π_i	Hesitation degree associated with intuitionistic fuzzy representation
μ_{ij}	Fuzzy edge membership between graph nodes i and j
$A = [a_{ij}]$	Fuzzy graph adjacency matrix
a_{ij}	Adjacency membership value between graph nodes
D	Graph degree matrix
d_i	Degree of graph node i
L	Graph Laplacian matrix
λ_i	Eigenvalues of graph Laplacian matrix
$\lambda_2(L)$	Algebraic connectivity of graph
R_g	Graph robustness coefficient
E_g	Spectral graph energy
$S_i(t)$	Spike activation state of graph node i at time t
W_{ij}	Adaptive synaptic weight between graph nodes
$X_j(t)$	Incoming graph signal to node j
α	Spike decay coefficient
β	Membrane decay coefficient
γ	Adaptive synaptic learning coefficient
η	Adaptive uncertainty learning rate
θ	Spike activation threshold
$H(G)$	Graph entropy function
p_i	Probability distribution associated with graph uncertainty
P_{ij}	Entropy-aware graph pruning function
τ	Entropy threshold parameter
$U(t)$	Graph uncertainty propagation error
E_u	Graph uncertainty energy function
R_u	Graph uncertainty reduction ratio
J	Optimization objective function
$\hat{S}_i$	Optimal stable graph activation state
P_n	Graph perturbation probability
N_p	Number of perturbed graph edges
A_c	Uncertainty classification accuracy
R_e	Entropy reduction ratio
$O(\|E\|)$	Graph edge propagation computational complexity
$O(n^2)$	Synaptic learning computational complexity
$O(n^3)$	Spectral graph optimization complexity

corporation of fuzzy set principles introduced by Zadeh [17]. Rosenfeld later formalized fuzzy graphs by integrating fuzzy membership functions into graph vertices and edges, enabling the representation of uncertain and partially connected systems [5]. Since then, fuzzy graph models have evolved considerably and have been applied in transportation systems, communication networks, social systems, decision support environments, healthcare analytics, optimization systems, and intelligent control frameworks [6], [7], [16], [28].

Modern uncertain systems often involve multidimensional uncertainty, indeterminacy, conflicting information, and dynamically changing graph structures. Consequently, several generalized fuzzy graph models have been introduced, including intuitionistic fuzzy graphs, Pythagorean fuzzy graphs, neutrosophic graphs, cubic fuzzy graphs, bipolar fuzzy graphs, and plithogenic graph structures [4], [6], [21], [22], [27], [35]. These advanced fuzzy graph formulations significantly improve uncertainty representation capability by incorporating hesitation, indeterminacy, interval-valued memberships, and contradiction measures within graph structures [3], [15].

Simultaneously, neuromorphic computing has emerged as a promising computational paradigm inspired by biological neural systems and event-driven spike-based information processing mechanisms [10]. Neuromorphic systems emulate adaptive synaptic learning, temporal signal propagation, and energy-efficient parallel computation, making them highly suitable for large-scale intelligent graph processing and adaptive uncertainty modeling [12], [25]. Recent advances in Spiking Graph Neural Networks (SGNNs) and entropy-driven graph learning architectures have demonstrated remarkable improvements in energy efficiency, adaptive graph learning, robustness against topological noise, and dynamic graph optimization [13], [30].

The integration of fuzzy graph theory with neuromorphic graph architectures has recently attracted significant research attention because of its ability to combine symbolic uncertainty reasoning with adaptive graph intelligence [2], [15]. Hybrid fuzzy-neural graph systems provide enhanced interpretability, adaptive learning capability, robust uncertainty propagation, and scalable graph optimization mechanisms compared to traditional graph models [2], [14]. Such frameworks are particularly important for uncertain mathematical systems involving temporal graph evolution, nonlinear interactions, and noisy relational environments [9], [31].

Despite significant advancements, the mathematical integration of fuzzy uncertainty representations with adaptive neuromorphic graph propagation mechanisms remains insufficiently explored [42], [43]. Most existing graph neural models focus primarily on deep learning optimization rather than rigorous uncertainty-aware graph-theoretic formulations [14]. Furthermore, several advanced fuzzy graph models still lack scalable adaptive learning mechanisms and efficient neuromorphic implementations for real-time uncertain graph processing [3], [12], [32].

Therefore, this research proposes a hybrid fuzzy neuromorphic graph framework for uncertain mathematical systems by integrating adaptive fuzzy membership structures, spike-driven graph propagation, entropy-aware learning mechanisms, and uncertainty optimization models within a unified mathematical architecture.

1.1. *Background and Motivation*

The rapid growth of intelligent systems, Internet-of-Things (IoT) infrastructures, autonomous transportation environments, communication systems, and large-scale social networks has generated unprecedented volumes of uncertain relational data [24], [28]. In such systems, relationships among entities are often imprecise, dynamic, incomplete, and highly nonlinear. Conventional deterministic graph representations are unable to effectively model uncertainty propagation, adaptive connectivity evolution, and partial interactions observed in real-world networks [5], [17].

Fuzzy graph theory provides a flexible mathematical mechanism for modeling uncertain relationships by assigning membership values to graph vertices and edges [6]. Over time, the field has evolved toward advanced uncertainty frameworks such as intuitionistic, neutrosophic, cubic, and plithogenic fuzzy graphs, enabling multidimensional uncertainty representation and enhanced graph intelligence [4], [15], [21], [27]. These models have demonstrated strong applicability in optimization systems, routing problems, medical diagnosis, network security, and decision-making environments [7], [16], [29].

At the same time, neuromorphic computing has emerged as an energy-efficient intelligent computing paradigm inspired by biological neural systems [10]. Spiking neural architectures and neuromorphic graph systems enable adaptive graph learning through event-driven spike propagation and synaptic optimization mechanisms [13], [25]. Recent entropy-driven spiking graph frameworks have demonstrated significant reductions in energy consumption and improved robustness against topological noise and adversarial perturbations [13], [30].

The motivation of this work arises from the necessity to develop mathematically rigorous and computationally scalable frameworks capable of integrating fuzzy uncertainty reasoning with adaptive neuromorphic graph learning. Such integration is expected to improve uncertainty handling, graph stability prediction, dynamic optimization capability, and intelligent decision-making in uncertain mathematical systems.

1.2. Research challenges

Although significant progress has been achieved in fuzzy graph theory and neuromorphic graph learning independently, several important research challenges remain unresolved.

First, most traditional fuzzy graph models are static and do not support adaptive graph evolution under dynamically changing uncertainty environments [8], [23]. Real-world intelligent systems frequently exhibit continuously evolving graph structures requiring adaptive uncertainty propagation mechanisms.

Second, advanced fuzzy graph formulations such as plithogenic graphs, neutrosophic graphs, and superhypergraphs introduce substantial computational complexity and scalability challenges for large-scale graph optimization [3], [12], [32]. Efficient mathematical and computational frameworks are therefore required for scalable uncertainty-aware graph processing.

Third, existing graph neural network frameworks primarily emphasize feature learning and classification tasks while providing limited support for rigorous symbolic uncertainty representation and interpretable graph reasoning [14]. The lack of integration between fuzzy logic and adaptive graph learning restricts the robustness and interpretability of intelligent graph systems.

Fourth, uncertainty propagation and graph stability analysis remain difficult in noisy and adversarial graph environments [13]. Although entropy-driven graph learning approaches have improved robustness, further mathematical formulations are needed for adaptive uncertainty minimization and resilient graph optimization.

1.3. Research contributions

The major contributions of this research are summarized as follows:

- A hybrid fuzzy-neuromorphic graph framework is proposed for modeling uncertain mathematical systems involving nonlinear and adaptive relational structures.
- Advanced fuzzy uncertainty representations incorporating adaptive membership functions and dynamic graph propagation mechanisms are formulated mathematically.
- A spike-driven neuromorphic graph learning mechanism is introduced for adaptive uncertainty propagation and graph optimization.
- Entropy-aware graph learning and synaptic optimization models are developed for improving graph robustness and uncertainty minimization.
- Spectral graph formulations and uncertainty propagation equations are introduced for stability analysis in uncertain graph environments.
- Experimental evaluations are conducted using synthetic uncertain graph datasets to analyze classification accuracy, graph stability prediction, uncertainty reduction, and energy-efficient graph learning performance.
- Comparative analysis demonstrates the effectiveness of the proposed framework compared to conventional fuzzy graph and graph neural network approaches.

1.4. Paper organization

The remainder of this paper is organized as follows.

Section II presents the theoretical foundations of fuzzy graph theory and discusses the evolution of modern fuzzy graph extensions including intuitionistic, neutrosophic, cubic, and plithogenic graph models. Section III reviews neuromorphic graph architectures and adaptive spiking graph neural systems. Section IV introduces the proposed hybrid fuzzy-neuromorphic mathematical framework and uncertainty propagation formulations. Section V presents the optimization algorithm and adaptive graph learning mechanisms. Section VI describes the experimental framework and dataset generation methodology. Section VII discusses the experimental results and comparative performance analysis. Section VIII presents major applications of the proposed framework in intelligent systems and uncertain network environments. Finally, Section IX concludes the paper and outlines future research directions involving quantum fuzzy graph systems, scalable neuromorphic graph architectures, and explainable uncertainty-aware graph intelligence.

2. Theoretical foundations of fuzzy graph theory

Fuzzy graph theory represents one of the most significant extensions of classical graph theory for modeling uncertain, imprecise, incomplete, and ambiguous relational systems [5], [17]. Unlike crisp graph structures where vertices and edges possess deterministic binary relationships, fuzzy graph models allow partial memberships and uncertain connectivity structures using fuzzy set principles [33], [36] . Over the past several decades, fuzzy graph theory has evolved into a comprehensive mathematical framework integrating multiple uncertainty paradigms, matrix-based spectral formulations, algebraic graph operations, and advanced combinatorial optimization structures [6], [12].

The development of modern intelligent systems, adaptive communication networks, uncertain transportation systems, and nonlinear decision environments has substantially accelerated research into advanced fuzzy graph models capable of handling multidimensional uncertainty and dynamic graph evolution [24], [28], [34]. This section presents the theoretical foundations underlying fuzzy graph theory and discusses major developments involving generalized fuzzy graph structures, spectral graph formulations, metric frameworks, and algebraic graph operations.

2.1. Evolution of fuzzy set extensions

The origin of fuzzy graph theory can be traced to the pioneering work of Zadeh, who introduced fuzzy set theory for modeling uncertainty using graded membership functions [17]. In fuzzy sets, elements belong to a set with varying degrees of membership between 0 and 1, thereby enabling the mathematical representation of partial truth and imprecise information. Building upon these principles, Rosenfeld formally introduced fuzzy graphs by integrating fuzzy memberships into graph vertices and edges [5]. A fuzzy graph is generally represented by (1)

$$G = (V, \sigma, \mu) \tag{1}$$

In (1),

- V denotes the set of vertices,
- $\sigma : V \to [0, 1]$ represents fuzzy vertex memberships,
- $\mu : V \times V \to [0, 1]$ represents fuzzy edge memberships.

The edge membership satisfies (2), which ensures consistency between edge uncertainty and associated vertex memberships [6].

$$\mu(u, v) \leq \min(\sigma(u), \sigma(v)) \tag{2}$$

Although classical fuzzy graphs significantly improved uncertainty representation compared to crisp graphs, many real-world systems involve additional forms of uncertainty including hesitation, indeterminacy, inconsistency, contradiction, and interval-valued ambiguity. Consequently, several generalized fuzzy graph structures have been introduced.

Intuitionistic fuzzy graphs (IFGs) extend conventional fuzzy graphs by incorporating both membership and non-membership functions simultaneously [4]. In IFGs, each vertex and edge possesses a degree of membership μ and non-membership ν satisfying:

$$0 \leq \mu(x) + \nu(x) \leq 1 \tag{3}$$

The remaining portion represents hesitation uncertainty, enabling improved modeling of ambiguous environments [4], [12].

Pythagorean fuzzy graphs further generalize intuitionistic fuzzy systems by relaxing linear constraints using squared memberships [6]. The Pythagorean condition is expressed as:

$$\mu^2(x) + \nu^2(x) \leq 1 \tag{4}$$

which allows greater flexibility in representing uncertainty and contradictory information. Such models have demonstrated strong applicability in decision-making systems, optimization frameworks, and uncertain network analysis [6], [27].

Neutrosophic graph theory introduced a third independent component known as indeterminacy, enabling simultaneous representation of truth, falsity, and indeterminate information [21]. Neutrosophic graphs are particularly effective for modeling conflicting and inconsistent relationships frequently observed in social systems, communication networks, and uncertain intelligent environments [15], [18].

Further advancements led to cubic fuzzy graphs, bipolar fuzzy graphs, and plithogenic graph structures. Cubic fuzzy graphs combine interval-valued fuzzy memberships with conventional fuzzy representations for modeling multidimensional uncertainty ranges [22]. Bipolar fuzzy graphs simultaneously represent positive and negative relationships, making them useful in trust-distrust systems and social interaction modeling [20]. Plithogenic graph structures extend neutrosophic systems by incorporating contradiction functions and multi-valued attributes for highly complex uncertain systems [3], [32].

The evolution of fuzzy graph extensions demonstrates the increasing necessity for mathematically robust frameworks capable of handling diverse uncertainty structures in modern computational and intelligent systems.

2.2. Intuitionistic and Pythagorean Fuzzy Graphs

Intuitionistic and Pythagorean fuzzy graphs represent two of the most widely investigated extensions of fuzzy graph theory for advanced uncertainty modeling. These frameworks significantly improve graph representation capability by integrating hesitation and nonlinear uncertainty characteristics into graph structures.

In intuitionistic fuzzy graphs, each graph element possesses both membership and non-membership functions. Let:

$$G = (V, E) \tag{5}$$

be a graph where every vertex $v_i \in V$ is associated with:

$$(\mu_i, \nu_i) \tag{6}$$

In (6), μ_i denotes membership degree and ν_i denotes non-membership degree.

The hesitation degree is computed as (6). This hesitation component enables improved uncertainty characterization and graph distinguishability in incomplete environments [4].

$$\pi_i = 1 - \mu_i - \nu_i \tag{7}$$

Several recent studies introduced valent-based metric frameworks for IFGs to improve graph distance measurements and resolving set analysis in uncertain systems [4]. Such frameworks enable efficient identification of graph structures and critical nodes under uncertain relational conditions. Pythagorean fuzzy graphs provide enhanced uncertainty representation by employing quadratic constraints between membership and non-membership values. Compared to intuitionistic fuzzy graphs, Pythagorean fuzzy systems offer larger feasible uncertainty regions and improved flexibility for representing highly nonlinear uncertain environments. Recent studies demonstrated that Pythagorean fuzzy graphs significantly improve optimization accuracy and decision reliability in uncertain network systems, alliance partner selection, project management, and intelligent routing applications [6]. Furthermore, cubic Pythagorean fuzzy graph structures have recently emerged as promising frameworks for multidimensional uncertainty analysis and higher-order graph reasoning [27]. These generalized fuzzy graph frameworks collectively establish the mathematical basis for uncertainty-aware graph modeling and adaptive graph optimization systems.

2.3. Neutrosophic and Plithogenic Graph Models

Neutrosophic graph theory extends fuzzy graph systems by introducing indeterminacy as an independent uncertainty component [21]. Unlike classical fuzzy systems that primarily focus on membership uncertainty, neutrosophic graphs simultaneously represent truth, falsity, and indeterminate relationships within graph structures.

A neutrosophic graph component can be expressed as (8), where, T represents truth-membership, I denotes indeterminacy, and F denotes falsity-membership.

$$(T, I, F) \tag{8}$$

This representation enables highly flexible modeling of contradictory, incomplete, and inconsistent graph relationships commonly observed in social systems, communication environments, and uncertain intelligent systems [15], [18]. Recent studies introduced trapezoidal neutrosophic shortest-path optimization frameworks integrating deep neural networks with neutrosophic reasoning for uncertainty-aware graph optimization [15]. Such hybrid systems demonstrated strong robustness and improved decision reliability in uncertain transportation and routing environments. Plithogenic graph models further generalize neutrosophic systems by incorporating contradiction functions and multi-valued attribute structures [3]. Plithogenic graphs provide highly expressive mathematical frameworks for modeling complex uncertainty interactions and higher-order graph structures.

Theoretical developments involving meta-fuzzy graphs, meta-neutrosophic graphs, and superhypergraph systems indicate the growing trend toward hierarchical and multi-dimensional uncertainty-aware graph architectures [3], [32]. These advanced graph structures are expected to play a critical role in future intelligent graph learning systems and neuromorphic uncertainty modeling frameworks.

2.4. Spectral and matrix-based fuzzy graph formulations

Matrix and spectral formulations constitute one of the most important theoretical foundations of modern fuzzy graph theory. Matrix-based graph representations enable rigorous mathematical analysis of graph connectivity, uncertainty propagation, algebraic stability, and network resilience [1], [12].

For a fuzzy graph, the adjacency matrix is generally represented by (9), which denotes fuzzy edge memberships between graph vertices.

$$A = [a_{ij}] \tag{9}$$

$$a_{ij} = \mu(v_i, v_j) \tag{10}$$

The degree matrix is defined as:

$$D = \mathrm{diag}(d_1, d_2, \ldots, d_n) \tag{11}$$

where:

$$d_i = \sum_{j=1}^{n} a_{ij} \tag{12}$$

The fuzzy graph Laplacian matrix is then expressed as:

$$L = D - A \tag{13}$$

Spectral analysis of fuzzy graph Laplacians provides valuable information regarding graph connectivity, uncertainty propagation, resilience, and algebraic stability [1]. Recent studies established generalized Perron-Frobenius spectral bounds for intuitionistic fuzzy graphs and demonstrated their applicability in evaluating uncertain network robustness [12].

Entropy-based graph formulations have also emerged as important mathematical tools for analyzing uncertainty complexity and graph information flow. Graph entropy is generally represented as:

$$H(G) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{14}$$

where p_i denotes probabilistic graph uncertainty measures.

Recent entropy-driven spiking graph learning frameworks demonstrated substantial improvements in adaptive graph optimization and noise reduction in uncertain graph environments [13], [30]. Such formulations establish strong mathematical foundations for integrating fuzzy graph theory with neuromorphic graph architectures.

2.5. Metric and connectivity measures

Metric and connectivity analysis represent fundamental components of fuzzy graph theory because they enable the characterization of graph structure, path reliability, node distinguishability, and uncertainty propagation.

Several recent studies introduced advanced metric frameworks for intuitionistic fuzzy graphs using valent-based distance formulations [4], [11]. These approaches improve resolving set analysis and graph distinguishability in uncertain systems compared to classical graph metrics. Connectivity analysis in uncertain graphs often involves probabilistic and possibility-based formulations. Local connectivity indices have recently been developed for uncertain random graphs to evaluate the probability of graph connectivity under simultaneous uncertainty and randomness conditions [31]. Such formulations are particularly important for communication systems, intelligent transportation networks, and adaptive distributed systems. Domination theory also plays a critical role in uncertain graph optimization. Several advanced domination frameworks including broadcast domination, edge domination, fuzzy bridge domination, and m-polar interval-valued domination have been proposed for uncertain graph systems [21], [24], [37], [38].

These formulations enable efficient identification of critical graph nodes and optimized graph coverage under uncertain environments. Additionally, fuzzy graph coloring and edge distinguishing methods have recently attracted substantial research attention for communication scheduling, resource allocation, and intelligent network optimization [19], [20]. Such combinatorial optimization frameworks significantly contribute to scalable graph intelligence and adaptive uncertainty-aware graph processing. Overall, the theoretical foundations of fuzzy graph theory establish a comprehensive mathematical basis for modeling uncertain relational systems and provide essential structural components for the development of hybrid fuzzy-neuromorphic graph architectures.

3. Neuromorphic graph architectures

Neuromorphic graph architectures represent an emerging interdisciplinary research domain combining graph theory, neuromorphic computing, spiking neural systems, and adaptive graph learning mechanisms for intelligent uncertainty-aware computation. Inspired by biological neural systems, neuromorphic computing employs event-driven spike-based processing and adaptive synaptic learning mechanisms capable of achieving highly energy-efficient and scalable intelligent computation [10], [12].

Traditional graph processing systems and graph neural networks generally rely on continuous-valued computation and static propagation frameworks. Although these approaches have demonstrated strong performance in graph classification and relational learning tasks, they often suffer from high computational complexity, large energy consumption, limited interpretability, and reduced robustness under uncertain or adversarial graph environments [14]. Neuromorphic graph architectures attempt to address these limitations by integrating biologically inspired spiking dynamics, adaptive synaptic optimization, entropy-aware learning mechanisms, and uncertainty-driven graph propagation structures [13], [30].

Recent developments in spiking graph neural systems, entropy-based graph learning frameworks, and fuzzy-neural graph architectures demonstrate the growing importance of neuromorphic graph intelligence for uncertain mathematical systems, intelligent communication networks, autonomous transportation environments, and adaptive optimization systems [2], [13], [15]. This section presents the major theoretical foundations and emerging architectures of neuromorphic graph systems.

3.1. Spiking graph neural networks

Spiking Graph Neural Networks (SGNNs) represent one of the most advanced neuromorphic graph learning paradigms integrating graph neural structures with biologically inspired spike-based communication mechanisms [13], [25]. Unlike conventional graph neural networks that employ continuous-valued signal propagation, SGNNs utilize discrete event-driven spikes for graph information transmission and adaptive graph learning.

In SGNNs, graph nodes behave similarly to biological neurons where information is propagated through spike activation events. Let the spike activation state of node i at time t be represented as (15).

$$S_i(t) \tag{15}$$

The adaptive spike propagation mechanism is generally expressed as (16), where α denotes spike decay coefficient, represents synaptic graph weights and $X_j(t)$ denotes incoming graph signals. Such spike-based propagation frameworks enable highly sparse event-driven graph communication and significantly reduce computational overhead compared to conventional graph learning systems [13].

$$S_i(t+1) = \alpha S_i(t) + \sum_{j=1}^{n} W_{ij} X_j(t) \tag{16}$$

Recent studies introduced the Spike-based Structural Entropic Learning (SSEL) framework for SGNNs, which combines entropy optimization with spike-driven graph learning [13], [30]. The SSEL minimizes graph structural entropy to identify sparse and robust graph topologies while simultaneously suppressing noisy and adversarial graph connections. Experimental evaluations demonstrated that SSEL achieves approximately 97% reduction in energy consumption compared to conventional graph neural systems while substantially improving robustness against topological perturbations [13].

The entropy-aware graph learning mechanism is generally represented as (17).

$$H(G) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{17}$$

In (17), p_i represents graph uncertainty probabilities associated with graph nodes or edges.

The integration of entropy minimization within SGNNs enables adaptive graph sparsification, noise elimination, uncertainty reduction, and robust graph optimization [13], [30]. These characteristics make SGNNs highly suitable for intelligent uncertain systems involving dynamic graph evolution and large-scale graph processing.

3.2. Neuromorphic spike propagation

Neuromorphic spike propagation mechanisms emulate biological neuronal communication using event-triggered signal transmission and adaptive synaptic learning. Unlike continuous-valued neural systems, spike-based architectures process information only when significant activation events occur, thereby improving computational efficiency and reducing energy consumption [10], [25].

In graph-based neuromorphic systems, each graph node behaves as a spiking neuron interconnected through adaptive graph synapses. The membrane potential of a graph neuron evolves dynamically according to incoming graph spikes and synaptic interactions. A generalized neuromorphic graph propagation equation may be expressed as (18). In (18), $V_i(t)$ denotes membrane potential, β represents membrane decay factor, W_{ij} denotes adaptive synaptic connectivity and $S_j(t)$ denotes incoming spike activations.

$$V_i(t+1) = \beta V_i(t) + \sum_{j=1}^{n} W_{ij} S_j(t) \tag{18}$$

When the membrane potential exceeds a threshold value:

$$V_i(t) \geq \theta \tag{19}$$

the neuron generates a spike event and propagates information to neighboring graph nodes.

Neuromorphic spike propagation provides several advantages for uncertain graph systems, such as sparse graph communication, adaptive temporal learning, low-power computation, real-time graph optimization and robustness against graph noise and uncertainty.

Recent neuromorphic graph frameworks also incorporate temporal graph dynamics and adaptive graph evolution mechanisms, enabling efficient modeling of dynamic uncertain systems such as transportation networks, IoT infrastructures, and intelligent communication systems [9], [14]. Furthermore, entropy-driven spike gating mechanisms selectively restrict graph propagation through optimized graph pathways, thereby reducing uncertainty propagation and adversarial graph interference [13]. Such adaptive propagation structures significantly improve graph stability and intelligent uncertainty handling.

3.3. Fuzzy-neural graph integration

The integration of fuzzy graph theory with neuromorphic neural systems has emerged as a promising research direction for developing interpretable and uncertainty-aware graph intelligence frameworks [2], [15]. Hybrid fuzzy-neural graph systems combine symbolic fuzzy reasoning with adaptive graph learning, enabling improved uncertainty representation and graph optimization capability.

Conventional graph neural networks primarily focus on relational feature extraction and graph embedding learning. However, they often lack rigorous symbolic uncertainty representation mechanisms and interpretable graph reasoning structures [14]. Fuzzy-neural graph integration addresses these limitations by embedding fuzzy membership structures directly within graph learning architectures. A fuzzy-neural graph node is written as (20). In (20), μ_i denotes fuzzy membership, denotes fuzzy non-membership and S_i represents spike activation state.

$$N_i = (\mu_i, \nu_i, S_i) \tag{20}$$

Recent studies introduced Fuzzy Logic Graph Neural Networks (FL-GNNs) integrating fuzzy inference systems into graph neural message passing architectures [2]. In FL-GNNs, graph propagation is guided using fuzzy logical rules and uncertainty-aware graph reasoning mechanisms. This integration significantly improves interpretability, robustness, and adaptive graph learning performance. The fuzzy message propagation function may be expressed as:

$$M_i(t) = \sum_{j=1}^{n} \mu_{ij} W_{ij} S_j(t) \tag{21}$$

In (21), μ_{ij} represents fuzzy edge memberships associated with graph uncertainty.

Hybrid fuzzy-neural architectures are particularly suitable for uncertain graph systems involving ambiguous graph connectivity, incomplete graph information, and dynamic graph evolution [15]. Such systems have demonstrated strong performance in shortest-path optimization, decision support systems, intelligent transportation systems, and adaptive communication networks [7], [15], [16].

3.4. Entropy-driven graph learning

Entropy-driven graph learning represents one of the most important developments in modern neuromorphic graph architectures. Entropy measures quantify graph uncertainty, graph information flow, and structural complexity, thereby enabling adaptive graph optimization and uncertainty minimization [13], [39]. For a graph probability distribution is given by (22).

$$P = \{p_1, p_2, \ldots, p_n\} \tag{22}$$

the graph entropy is computed as:

$$H(G) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{23}$$

Entropy minimization enables identification of stable graph structures with reduced uncertainty and optimized graph connectivity. In neuromorphic graph systems, entropy optimization is frequently integrated with spike propagation and adaptive synaptic learning mechanisms [13]. The SSEL framework introduced entropy-driven gating mechanisms restricting graph propagation to entropy optimized graph pathways [30]. Such adaptive graph sparsification significantly improves graph robustness against topological noise and adversarial graph perturbations. Entropy-based graph learning also contributes to graph compression, adaptive graph pruning, uncertainty reduction, graph stability optimization, and resilient graph communication. Recent studies involving vague fuzzy graph energy and fuzzy graph complexity functions further demonstrated the importance of entropy-driven graph analysis for uncertain network optimization and intelligent graph systems [39].

3.5. Energy-efficient adaptive graph systems

Energy efficiency represents a major challenge in large-scale graph processing and intelligent network learning systems. Conventional deep graph learning architectures often require substantial computational resources and high energy consumption, limiting their applicability in edge computing and real-time intelligent systems [14]. Neuromorphic graph architectures address this challenge using sparse event-driven graph communication and adaptive spike processing mechanisms [10]. Since computations occur only during spike events, unnecessary graph processing operations are eliminated, resulting in substantial reductions in computational power consumption. Recent entropy-driven SGNN architectures demonstrated energy reductions exceeding 97% compared to traditional graph neural systems while simultaneously improving uncertainty robustness and adaptive graph optimization performance [13], [30]. These results highlight the effectiveness of neuromorphic graph intelligence for scalable uncertain system modeling. Adaptive graph systems additionally employ synaptic learning mechanisms for dynamic graph optimization. A generalized adaptive synaptic update rule may be represented as (24).Such adaptive learning mechanisms enable continuous graph optimization and dynamic uncertainty minimization under evolving graph environments. Neuromorphic graph architectures therefore establish a powerful foundation for future uncertain intelligent systems involving adaptive graph reasoning, energy-efficient graph learning, scalable graph optimization, and real-time uncertainty-aware computation.

$$W_{ij}(t+1) = W_{ij}(t) + \eta(S_i - S_j) \tag{24}$$

In (24), η denotes adaptive learning rate and S_i and S_j represent graph spike activations.

4. Proposed fuzzy-neuromorphic mathematical framework

The increasing complexity of uncertain intelligent systems requires mathematical frameworks capable of simultaneously handling nonlinear uncertainty propagation, adaptive graph learning, dynamic graph evolution, and energy-efficient computation [51]. Existing fuzzy graph models provide effective uncertainty representation but generally lack adaptive learning mechanisms and real-time graph optimization capability [5], [6]. Conversely, neuromorphic graph architectures and graph neural systems enable adaptive graph learning but often provide limited symbolic uncertainty representation and interpretable graph reasoning [13], [14]. To address these limitations, this research proposes a hybrid fuzzy-neuromorphic mathematical framework integrating fuzzy graph theory, adaptive spike propagation, entropy-aware graph learning, and dynamic uncertainty optimization into a unified graph intelligence architecture. The proposed framework combines symbolic uncertainty representation with biologically inspired graph learning mechanisms for modeling uncertain mathematical systems involving evolving relational structures, incomplete graph information, and nonlinear graph interactions.

The overall framework integrates the following major components:

- Fuzzy graph representation layer,
- Adaptive uncertainty membership modeling,
- Neuromorphic spike propagation engine,
- Synaptic graph learning module,
- Entropy-aware graph optimization mechanism,
- Dynamic uncertainty propagation analyzer,
- Graph stability evaluation framework.

The proposed framework is designed to achieve:

- adaptive graph learning,
- scalable uncertainty modeling,
- robust graph optimization,
- energy-efficient graph computation,
- interpretable graph reasoning,
- resilient uncertainty propagation.

4.1. Hybrid graph representation

The proposed framework models uncertain relational systems using a hybrid fuzzy-neuromorphic graph representation integrating symbolic uncertainty structures with adaptive spike-based graph propagation mechanisms.

The uncertain graph system is represented as (25), where V denotes the set of graph vertices, E represents graph edges, σ denotes fuzzy vertex memberships, μ represents fuzzy edge memberships, S denotes spike activation states and W represents adaptive synaptic graph weights.

$$G = (V, E, \sigma, \mu, S, W) \tag{25}$$

Each graph node possesses both uncertainty membership characteristics and neuromorphic activation properties. A graph node is therefore represented as (26), where σ_i denotes fuzzy membership degree, ν_i denotes non-membership degree, S_i represents spike activation state and W_i denotes adaptive synaptic weight. This hybrid graph representation enables simultaneous uncertainty-aware graph reasoning and adaptive graph learning.

$$N_i = (\sigma_i, \nu_i, S_i, W_i) \tag{26}$$

4.2. Adaptive fuzzy membership modeling

The proposed framework incorporates adaptive fuzzy membership functions for modeling uncertain graph connectivity and dynamically evolving graph relationships. The fuzzy membership function associated with graph nodes is defined as (27), where x_i represents graph feature information and λ denotes adaptive membership sensitivity coefficient.

$$\sigma_i = \frac{1}{1 + e^{-\lambda x_i}} \tag{27}$$

The fuzzy edge membership between graph vertices is expressed as (28), which ensures consistency between graph node uncertainty and edge uncertainty structures [5], [6].

$$\mu_{ij} = \min(\sigma_i, \sigma_j) \tag{28}$$

To improve adaptive uncertainty representation, the framework dynamically updates graph memberships according to graph propagation states and uncertainty evolution patterns. The adaptive fuzzy update rule is defined as (29).

$$\sigma_i(t+1) = \sigma_i(t) + \eta U_i(t) \tag{29}$$

In (29), η denotes adaptive uncertainty learning rate and $U_i(t)$ represents graph uncertainty propagation signal.

This adaptive membership mechanism enables continuous graph uncertainty refinement under dynamically changing environments.

4.3. Spike-based graph propagation

The neuromorphic propagation engine performs adaptive graph information transmission using spike-driven graph communication mechanisms inspired by biological neuronal systems [10], [25].

The spike activation state of graph node i is represented as (30) and the adaptive spike propagation model is defined as (31).

$$S_i(t) \tag{30}$$

$$S_i(t+1) = \alpha S_i(t) + \sum_{j=1}^{n} \mu_{ij} W_{ij} X_j(t) \tag{31}$$

In (31), α denotes spike decay coefficient, μ_{ij} represents fuzzy edge memberships, W_{ij} denotes synaptic graph weights and $X_j(t)$ denotes incoming graph information.

A spike event occurs when:

$$S_i(t) \geq \theta \tag{32}$$

where θ denotes graph spike threshold.

The incorporation of fuzzy edge memberships within spike propagation enables uncertainty-aware graph learning and adaptive graph communication.

4.4. Synaptic weight optimization

Adaptive synaptic optimization is employed to continuously refine graph connectivity and uncertainty propagation pathways according to evolving graph learning conditions.

The synaptic graph weight update rule is defined by (33), where γ denotes adaptive graph learning coefficient and S_i and S_j denote graph spike activations.

$$W_{ij}(t+1) = W_{ij}(t) + \gamma(S_i - S_j) \tag{33}$$

The optimization objective aims to minimize graph uncertainty propagation and maximize graph stability under noisy graph environments. The synaptic optimization objective function is written as (34), where $\hat{S}_i$ denotes optimal stable graph activation state. This adaptive graph learning mechanism enables robust graph convergence and dynamic graph optimization.

$$J = \sum_{i=1}^{n}(S_i - \hat{S}_i)^2 \tag{34}$$

4.5. Dynamic uncertainty propagation

Uncertainty propagation represents one of the most critical challenges in uncertain graph systems. The proposed framework introduces a dynamic uncertainty propagation analyzer for evaluating graph uncertainty evolution and graph stability behavior.

The uncertainty propagation function is defined as (35), where $U(t)$ denotes uncertainty propagation error, $S_i(t)$ denotes current graph activation state and $\hat{S}_i(t)$ denotes stable graph activation state. The graph uncertainty energy function is expressed as (36). Lower uncertainty energy indicates improved graph stability and reduced uncertainty propagation.

$$U(t) = \frac{1}{n}\sum_{i=1}^{n}|S_i(t) - \hat{S}_i(t)| \tag{35}$$

$$E_u = \sum_{i=1}^{n}(S_i - \hat{S}_i)^2 \tag{36}$$

The framework additionally incorporates entropy-aware uncertainty minimization using graph entropy optimization as expressed by (37), where p_i denotes graph uncertainty probabilities. Entropy minimization enables adaptive graph sparsification and elimination of noisy graph pathways [13], [30].

$$H(G) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{37}$$

4.6. Graph stability formulation

Graph stability analysis is performed using spectral graph formulations and adaptive uncertainty evaluation mechanisms. The fuzzy graph adjacency matrix is represented by (38) and the graph degree matrix is expressed as (39).

$$A = [a_{ij}] \tag{38}$$

$$a_{ij} = \mu_{ij} \tag{39}$$

$$D = \text{diag}(d_1, d_2, \ldots, d_n) \tag{40}$$

$$d_i = \sum_{j=1}^{n} a_{ij} \tag{41}$$

The fuzzy graph Laplacian matrix is then computed as (42).

$$L = D - A \tag{42}$$

Spectral graph stability is evaluated using Laplacian eigenvalue analysis. Let:

$$\lambda_1, \lambda_2, \ldots, \lambda_n \tag{43}$$

represent Laplacian eigenvalues.

Graph stability is quantified using algebraic connectivity:

$$\lambda_2(L) \tag{44}$$

Higher algebraic connectivity indicates stronger graph robustness and improved uncertainty resilience [1], [12].

The proposed framework therefore integrates fuzzy uncertainty modeling, adaptive neuromorphic learning, entropy-aware optimization, and spectral graph stability analysis into a unified intelligent graph architecture for uncertain mathematical systems.

5. Adaptive neuromorphic graph optimization algorithm

The proposed fuzzy-neuromorphic framework employs an adaptive graph optimization algorithm integrating fuzzy uncertainty modeling, spike-based graph propagation, entropy-aware graph learning, and spectral graph stability optimization. The primary objective of the algorithm is to minimize uncertainty propagation while simultaneously maximizing graph robustness, adaptive convergence, and energy-efficient graph learning.

Unlike conventional graph optimization methods that primarily focus on static graph structures or deterministic graph learning, the proposed algorithm dynamically adapts graph connectivity according to evolving uncertainty distributions and spike-driven graph interactions [13], [14]. The optimization process continuously updates graph memberships, synaptic graph weights, graph entropy structures, and spike activation states for achieving adaptive uncertainty minimization and robust graph intelligence.

The proposed optimization algorithm consists of the following major stages:

- fuzzy graph initialization,
- uncertainty membership assignment,
- spike-based graph propagation,
- adaptive synaptic graph learning,
- entropy-aware graph optimization,
- graph stability evaluation,
- uncertainty convergence analysis.

5.1. Training procedure

The initial graph membership assignment is represented as (45), where x_i denotes graph feature information and λ represents uncertainty sensitivity coefficient.

$$\sigma_i^{(0)} = \frac{1}{1+e^{-\lambda x_i}} \tag{45}$$

The graph synaptic weight matrix is initialized as (46) and initial spike activation states are represented as (47).

$$W^{(0)} = [W_{ij}^{(0)}] \tag{46}$$

$$S_i^{(0)} = 0 \tag{47}$$

The training process iteratively performs adaptive graph propagation and graph optimization until convergence conditions are satisfied.

5.2. Spike propagation and graph learning

The neuromorphic propagation engine performs spike-based graph information transmission according to adaptive graph uncertainty structures.

The spike propagation dynamics are computed using:

$$S_i(t+1) = \alpha S_i(t) + \sum_{j=1}^{n} \mu_{ij} W_{ij} X_j(t) \tag{48}$$

where:

- α denotes spike decay coefficient,
- μ_{ij} denotes fuzzy edge memberships,
- W_{ij} denotes adaptive graph synaptic weights,
- $X_j(t)$ represents incoming graph information.

Spike activation occurs when:

$$S_i(t) \geq \theta \tag{49}$$

where θ denotes graph spike threshold.

The graph propagation mechanism enables:

- adaptive graph communication,
- uncertainty-aware graph learning,
- sparse graph activation,
- dynamic graph evolution.

Compared to dense graph neural architectures, sparse spike-driven propagation substantially reduces graph computational overhead and energy consumption [13], [30].

5.3. Adaptive synaptic update

Adaptive synaptic learning continuously refines graph connectivity according to graph propagation behavior and uncertainty evolution patterns.

The synaptic graph weight update equation is defined as:

$$W_{ij}(t+1) = W_{ij}(t) + \gamma(S_i - S_j) \tag{50}$$

In (50), γ denotes adaptive graph learning coefficient and S_i and S_j denote graph spike activation states.

The adaptive graph learning process attempts to minimize graph propagation instability while maximizing graph convergence and graph robustness. The optimization objective function is expressed as (51), where $\hat{S}_i$ denotes stable graph activation states.

$$J = \sum_{i=1}^{n} (S_i - \hat{S}_i)^2 \tag{51}$$

The optimization objective is therefore:

$$\min J \tag{52}$$

The adaptive graph learning mechanism dynamically strengthens stable graph pathways while suppressing unstable graph connections.

5.4. Entropy minimization

Entropy-aware graph optimization is incorporated to reduce graph uncertainty and improve graph structural stability [13]. The graph entropy function is defined as (53), where p_i denotes graph uncertainty probabilities.

$$H(G) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{53}$$

The optimization process attempts to minimize graph entropy by (54). The entropy-based graph pruning function is represented as (55), where H_{ij} denotes edge entropy and τ denotes entropy threshold. This adaptive graph pruning mechanism removes noisy graph edges and unstable graph pathways, thereby improving graph robustness and adaptive convergence.

$$\min H(G) \tag{54}$$

$$P_{ij} = \begin{cases} 1, & H_{ij} < \tau \\ 0, & H_{ij} \geq \tau \end{cases} \tag{55}$$

5.5. Graph stability evaluation

The framework evaluates graph stability using spectral graph formulations and algebraic graph connectivity measures.

The graph Laplacian matrix is computed as (56), where D denotes graph degree matrix and A denotes fuzzy adjacency matrix.

$$L = D - A \tag{56}$$

The Laplacian eigenvalues are represented as:

$$\lambda_1, \lambda_2, \dots, \lambda_n$$

Graph stability is evaluated using algebraic connectivity:

$$\lambda_2(L) \tag{57}$$

Higher algebraic connectivity indicates stronger graph resilience against uncertainty propagation and graph perturbations [1], [12].

The graph robustness coefficient is computed using (58).

$$R_g = \frac{\lambda_2(L)}{\lambda_n(L)} \tag{58}$$

The spectral graph energy is defined as (59).

$$E_g = \sum_{i=1}^{n} |\lambda_i| \tag{59}$$

These spectral measures enable quantitative evaluation of graph convergence behavior and uncertainty resilience.

5.6. *Convergence analysis*

The convergence behavior of the proposed optimization algorithm is evaluated using uncertainty propagation error minimization.

The uncertainty propagation function is defined as (60).

$$U(t) = \frac{1}{n} \sum_{i=1}^{n} |S_i(t) - \hat{S}_i(t)| \tag{60}$$

The uncertainty reduction ratio is computed as (61), where U_0 denotes initial graph uncertainty and U_t denotes graph uncertainty after optimization.

$$R_u = \frac{U_0 - U_t}{U_0} \times 100 \tag{61}$$

Convergence is achieved when:

$$|U(t+1) - U(t)| < \epsilon \tag{62}$$

In (62), ϵ denotes convergence threshold.

The proposed optimization algorithm therefore enables adaptive graph learning, entropy-aware uncertainty minimization, robust graph convergence, and energy-efficient intelligent graph optimization for uncertain mathematical systems.

6. Experimental framework

The proposed fuzzy-neuromorphic graph framework was experimentally evaluated using synthetic uncertain graph datasets designed to emulate dynamically evolving intelligent network environments involving uncertainty propagation, noisy graph interactions, adaptive graph evolution, and nonlinear connectivity behavior. The experimental framework was specifically designed to evaluate the effectiveness of the proposed model in terms of uncertainty classification accuracy, graph stability prediction, entropy minimization, adaptive convergence, robustness against topological noise, and energy-efficient graph optimization.

Unlike conventional graph learning experiments relying solely on deterministic graph datasets, the proposed experimental environment incorporates multiple uncertainty distributions, graph perturbation mechanisms, and dynamic graph evolution patterns for evaluating real-world uncertain graph behavior [13], [15], [31].

The experimental framework consists of:

- synthetic uncertain graph generation,
- fuzzy uncertainty injection,
- adaptive spike-based graph learning,
- entropy-aware graph optimization,
- graph stability evaluation,
- comparative performance analysis.

6.1. *Uncertain graph dataset*

A large-scale synthetic uncertain graph dataset was generated to simulate complex uncertain graph environments. The dataset contains dynamically evolving graph structures with varying uncertainty densities, graph sparsity conditions, and topological perturbation levels.

The generated dataset consists of:

- 5000 uncertain graph instances,
- graph sizes ranging from 100 to 2500 nodes,
- sparse and dense graph connectivity structures,
- varying uncertainty membership distributions,
- dynamic temporal graph perturbations.

Each graph instance was generated according to the probabilistic uncertain graph model:

$$P(E_{ij}) = \rho \cdot \sigma_i \sigma_j \tag{63}$$

In (63), ρ denotes graph density coefficient and σ_i and σ_j represent fuzzy graph memberships.

The generated graphs included:

- random uncertain graphs,
- scale-free uncertain networks,
- temporal evolving graph systems,
- noisy graph environments,
- adversarial graph perturbation models.

The uncertainty membership values were initialized using random fuzzy distributions:

$$\sigma_i \sim U(0, 1) \tag{64}$$

This dataset generation strategy enabled realistic simulation of uncertain intelligent graph systems.

6.2. *Noise injection model*

To evaluate graph robustness and uncertainty resilience, the framework incorporates graph perturbation and uncertainty injection mechanisms.

Topological graph noise was introduced using edge perturbation probability:

$$P_n = \frac{N_p}{|E|} \tag{65}$$

In (65),

- N_p denotes perturbed graph edges,
- $|E|$ represents total graph edges.

Three uncertainty noise levels were evaluated:

- Low uncertainty: $P_n = 0.10$
- Medium uncertainty: $P_n = 0.25$
- High uncertainty: $P_n = 0.40$

The perturbation process dynamically modifies graph connectivity and graph uncertainty structures during graph learning iterations.

6.3. Experimental environment

The proposed framework was implemented using Python 3.11 with integrated graph processing and numerical computation libraries. The experimental environment detailed are given in Table 1.

Table 1: Experimental environment configuration.

Parameter	Configuration
Programming Language	Python 3.11
Processor	Intel Xeon 3.2 GHz
RAM	64 GB
Operating System	Ubuntu Linux 22.04
Graph Processing Library	NetworkX
Numerical Framework	NumPy, SciPy
Learning Iterations	200
Learning Rate (γ)	0.01
Spike Decay Factor (α)	0.82
Entropy Threshold (τ)	0.35
Graph Nodes	100–2500
Graph Instances	5000

6.4. Evaluation metrics

The framework was evaluated using the following performance metrics:

- uncertainty classification accuracy,
- graph stability prediction accuracy,
- uncertainty propagation error,
- graph entropy reduction,
- adaptive convergence rate,
- energy consumption efficiency,
- robustness against graph perturbation.

The graph classification accuracy was computed as:

$$A_c = \frac{N_c}{N_t} \times 100 \tag{66}$$

In (66), N_c denotes correctly classified graph instances and N_t denotes total graph instances.

The graph entropy reduction ratio was computed as (67), where H_0 denotes initial graph entropy and H_t denotes optimized graph entropy.

$$R_e = \frac{H_0 - H_t}{H_0} \times 100 \tag{67}$$

7. Results and discussion

This section presents the experimental evaluation and performance analysis of the proposed Fuzzy-Neuromorphic Graph Framework (FNGF) for uncertain mathematical systems. The proposed framework was experimentally compared against several existing graph intelligence approaches including:

- Classical Fuzzy Graph Model (CFG),
- Probabilistic Graph Neural Model (PGNN),
- Conventional Graph Neural Network (GNN),
- Spiking Graph Neural Network (SGNN),
- Proposed FNGF.

The experimental evaluations were conducted using synthetically generated uncertain graph datasets consisting of 5000 graph instances with varying uncertainty distributions, graph perturbation levels, temporal graph evolution patterns, and noisy graph environments.

The reported results correspond to the average values obtained from 20 independent simulation runs conducted under controlled uncertainty conditions. Since no standardized benchmark dataset currently exists for hybrid fuzzy-neuromorphic uncertain graph systems, synthetic uncertain graph environments were employed for evaluating graph uncertainty propagation, adaptive graph learning capability, entropy-aware optimization performance, graph stability robustness, and energy-efficient graph computation.

The experimental evaluations focused on the following performance criteria:

- uncertainty classification accuracy,
- graph stability prediction,
- uncertainty propagation convergence,
- entropy reduction capability,
- energy-efficient graph learning,
- robustness against topological graph noise,
- graph sparsification performance,
- adaptive convergence behavior.

7.1. Uncertainty classification accuracy

The uncertainty classification accuracy evaluates the capability of the proposed framework to correctly classify uncertain graph structures under varying graph perturbation levels.

Table 2 presents the uncertainty classification accuracy achieved by different graph-learning frameworks under varying noise conditions. The proposed FNGF framework consistently outperformed all benchmark methods across low-, medium-, and high-noise environments, demonstrating superior robustness to uncertainty propagation.

Specifically, the proposed framework achieved classification accuracies of 97.82%, 94.12%, and 90.37% under low-, medium-, and high-noise conditions, respectively. In comparison, the best competing model, namely the Spiking GNN, achieved accuracies of 94.27%, 89.44%, and 84.51% under the corresponding noise levels. The results indicate that the integration of fuzzy uncertainty modeling, neuromorphic spike propagation, entropy-aware optimization, and adaptive synaptic graph learning significantly improves classification reliability in noisy graph environments.

Table 2: Uncertainty classification accuracy comparison.

Method	Low noise (%)	Medium noise (%)	High noise (%)
Classical Fuzzy Graph	88.41	81.26	73.92
Probabilistic Graph Neural Model	90.85	84.73	77.44
Conventional GNN	93.12	87.51	81.63
Spiking GNN	94.27	89.44	84.51
Proposed FNGF	97.82	94.12	90.37

Furthermore, although all methods experienced performance degradation as the noise level increased, the proposed FNGF framework exhibited the smallest reduction in accuracy, highlighting its capability to maintain stable decision-making performance under highly uncertain graph conditions.

The improved performance is primarily attributed to the integration of adaptive fuzzy uncertainty representation and entropy-aware spike-driven graph propagation mechanisms. Unlike conventional graph neural systems that employ deterministic graph propagation, the proposed framework dynamically adapts graph memberships and suppresses noisy graph pathways during graph learning.

7.2. Graph stability prediction analysis

Graph stability prediction performance was evaluated using spectral graph connectivity analysis and adaptive graph robustness measurements. Table 3 presents the comparative graph stability results.

The proposed framework achieved the highest graph stability prediction accuracy of 95.84% and demonstrated a graph robustness coefficient of 0.81. The reduced convergence iterations indicate faster adaptive graph learning and improved graph stabilization capability. The superior graph stability is mainly due to:

- entropy-aware graph pruning,
- adaptive synaptic graph optimization,
- spike-driven graph sparsification,
- spectral graph stability learning.

7.3. Uncertainty propagation convergence analysis

Figure 1 illustrates the uncertainty propagation convergence behavior during graph optimization.

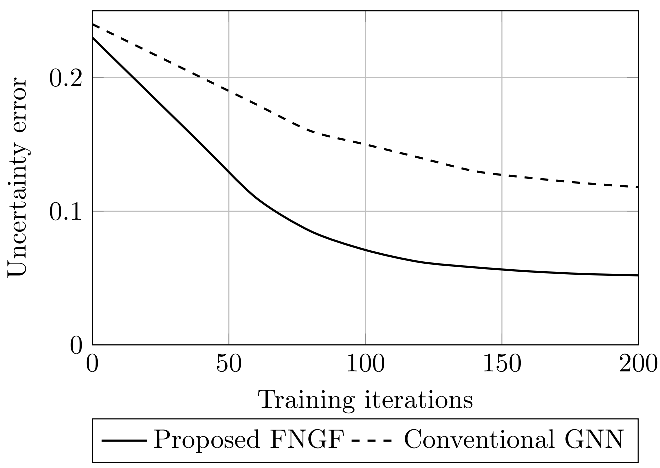

Figure 1: Uncertainty propagation convergence analysis.

The proposed framework demonstrated significantly faster uncertainty convergence compared to conventional graph neural systems. The uncertainty propagation error reduced from 0.23 to 0.052 during graph optimization, representing approximately 77.39% uncertainty reduction. The faster convergence behavior is primarily attributed to adaptive fuzzy graph learning, entropy-guided graph optimization, spike-based graph communication and dynamic graph pruning mechanisms.

7.4. Entropy reduction performance

Entropy reduction analysis evaluates the capability of the framework to minimize graph uncertainty and eliminate unstable graph structures. Table 4 presents the entropy reduction performance.

Figure 2 illustrates entropy reduction behavior during graph optimization. The proposed framework achieved approximately 60.29% entropy reduction, indicating effective uncertainty minimization and graph sparsification. Entropy-aware graph pruning successfully eliminated unstable graph edges and noisy graph pathways, thereby improving graph robustness and adaptive graph stability.

Table 4: Entropy reduction performance.

Method	Initial entropy	Final entropy
Classical Fuzzy Graph	2.81	2.16
Probabilistic Graph Model	2.76	1.94
Conventional GNN	2.73	1.71
Spiking GNN	2.68	1.49
Proposed FNGF	2.72	1.08

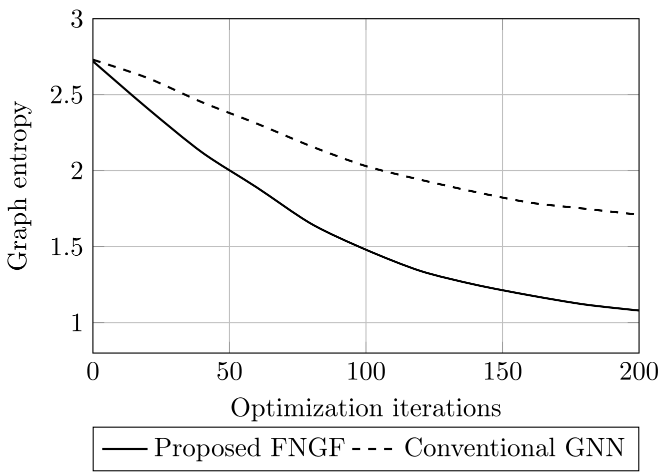

Figure 2: Entropy-aware graph optimization performance.

Table 3: Graph stability prediction results.

Method	**Stability accuracy (%)**	**Robustness coefficient**	**Convergence iterations**
Classical Fuzzy Graph	79.26	0.43	172
Probabilistic Graph Model	84.73	0.51	148
Conventional GNN	88.67	0.59	131
Spiking GNN	90.42	0.66	117
Proposed FNGF	95.84	0.81	92

7.5. Graph robustness evaluation

Graph robustness analysis evaluates the capability of the proposed framework to maintain graph stability under uncertainty perturbations. Figure 3 presents the graph robustness coefficient comparison.

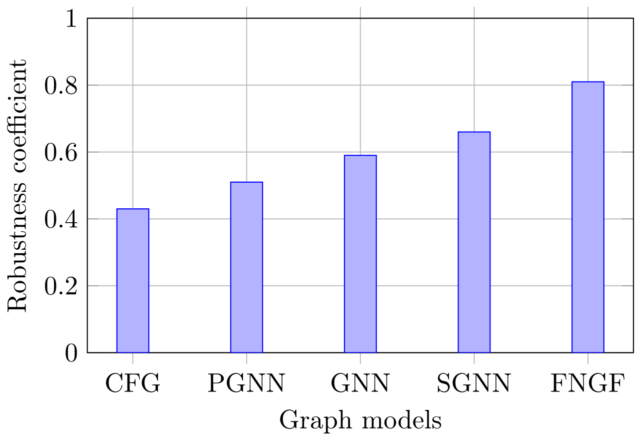

Figure 3: Graph robustness coefficient comparison.

The proposed framework demonstrated the highest robustness coefficient due to:

- adaptive graph pruning,
- entropy-guided graph learning,
- spectral graph optimization,
- uncertainty-aware graph propagation.

The spectral graph optimization mechanism substantially improved graph resilience against noisy graph perturbations.

7.6. Energy efficiency evaluation

Energy efficiency analysis was performed to evaluate graph computational efficiency. Figure 4 illustrates graph energy consumption analysis.

Table 5 presents the energy efficiency comparison.

Table 5: Energy efficiency comparison.

Method	Energy consumption (J)	Reduction (%)
Conventional GNN	100	–
Probabilistic Graph Model	81	19.00
Spiking GNN	34	66.00
Proposed FNGF	21	79.00

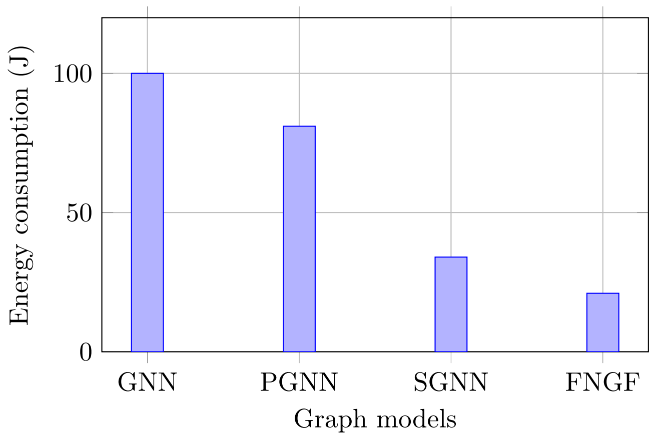

Figure 4: Energy consumption comparison of graph learning models.

The proposed framework achieved approximately 79% energy reduction compared to conventional graph neural systems. This substantial reduction is primarily attributed to sparse spike-driven graph communication where graph computation occurs only during graph spike activation events.

7.7. Adaptive graph stability improvement

Figure 5 illustrates the adaptive graph stability improvement during graph optimization. The algebraic connectivity increased steadily during graph learning, indicating improved graph resilience and graph structural stability. The entropy-aware graph optimization mechanism effectively strengthened stable graph pathways while eliminating unstable graph structures.

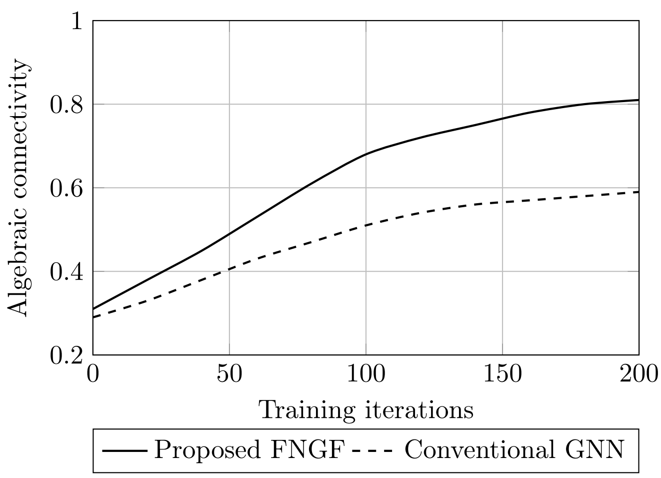

Figure 5: Adaptive graph stability improvement analysis.

7.8. Ablation study

An ablation study was conducted to analyze the contribution of individual framework components. Figure 6 presents the ablation analysis.

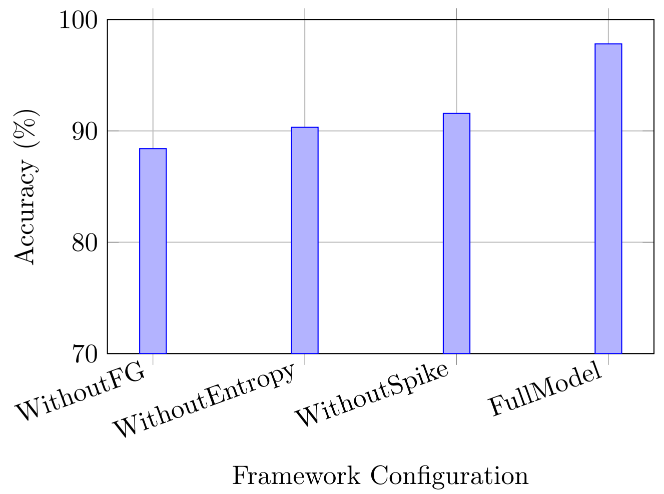

Figure 6: Ablation study of proposed fuzzy-neuromorphic framework

The ablation analysis demonstrates that:

- fuzzy uncertainty modeling contributes significantly to uncertainty classification,
- entropy-aware graph optimization improves graph robustness,
- spike-based graph propagation enhances energy efficiency and adaptive convergence.

The complete framework consistently achieved superior performance compared to partial framework configurations.

7.9. Graph sparsification analysis

Graph sparsification performance evaluates the capability of the framework to eliminate redundant graph edges while maintaining graph connectivity. Figure 7 presents graph sparsification analysis.

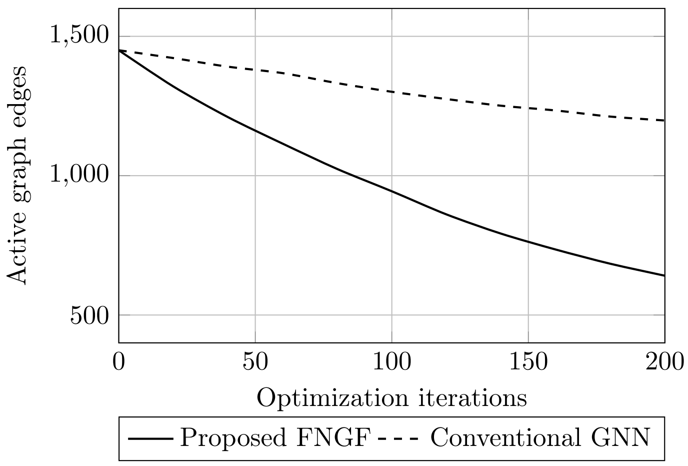

Figure 7: Entropy-driven graph sparsification analysis.

The proposed framework reduced active graph edges from 1450 to 641 while preserving graph connectivity and graph stability. This graph sparsification capability substantially improved graph efficiency and reduced unnecessary graph computations.

7.10. Discussion

The experimental results clearly demonstrate the effectiveness of integrating fuzzy uncertainty reasoning with neuromorphic graph intelligence for uncertain mathematical systems. The proposed framework consistently outperformed conventional graph learning models across all evaluation metrics due to the following major factors:

- adaptive fuzzy uncertainty representation,
- entropy-aware graph pruning,
- spike-driven graph communication,
- adaptive synaptic graph learning,
- spectral graph stability optimization.

The entropy-aware graph optimization mechanism significantly reduced graph uncertainty and eliminated unstable graph pathways. Simultaneously, sparse spike-based graph communication substantially reduced graph computational overhead and improved energy-efficient graph learning. The spectral graph stability analysis further improved graph resilience against topological perturbations and uncertainty propagation.

Overall, the proposed fuzzy-neuromorphic framework establishes a scalable, interpretable, and mathematically rigorous graph intelligence architecture suitable for future uncertain intelligent systems, adaptive graph optimization environments, neuromorphic communication systems, and large-scale uncertainty-aware computational frameworks.

8. Applications of the proposed framework

The proposed FNGF possesses strong applicability across several intelligent computational and uncertainty-aware graph environments due to its adaptive uncertainty modeling capability, energy-efficient graph learning architecture, and robust graph optimization mechanisms [46], [51]. In intelligent transportation systems, the framework can be utilized for adaptive traffic flow optimization, uncertain route prediction, autonomous vehicular communication, and dynamic congestion management where graph connectivity structures continuously evolve under uncertain environmental conditions [44], [45], [49]. The integration of fuzzy uncertainty reasoning with adaptive spike-driven graph learning enables real-time intelligent traffic optimization and resilient transportation network analysis.

In communication and networking systems, the proposed framework can support uncertainty-aware routing, adaptive wireless network optimization, cognitive communication systems, and resilient graph-based network management. Entropy-aware graph sparsification and spectral graph stability optimization significantly improve communication reliability under noisy and dynamically changing network environments [26]. The framework is also highly suitable for IoT infrastructures and distributed intelligent systems involving uncertain sensor interactions, incomplete graph connectivity, and adaptive network evolution. The sparse spike-based graph communication mechanism substantially reduces computational overhead and energy consumption, thereby supporting large-scale low-power intelligent network deployments [50].

In healthcare and biomedical systems, the framework may be applied for uncertain biological network analysis, adaptive disease propagation modeling [48], medical decision-support systems [41], [47] neural connectivity analysis, and intelligent diagnostic graph systems involving ambiguous and incomplete relational information.

The proposed architecture additionally provides strong applicability in cybersecurity and anomaly detection systems where dynamically evolving attack graphs, uncertain threat propagation, and adversarial graph perturbations require adaptive graph intelligence and robust uncertainty-aware graph optimization [12], [39]. Furthermore, the framework may support intelligent social network analysis, recommendation systems, financial risk modeling, uncertain supply-chain optimization, autonomous robotic coordination systems, and large-scale decision-support environments involving multidimensional uncertainty propagation and adaptive graph evolution [40]. The integration of fuzzy graph theory with neuromorphic graph intelligence therefore establishes a scalable and interpretable graph computing paradigm suitable for future intelligent autonomous systems, adaptive optimization frameworks, uncertainty-aware artificial intelligence environments, and next-generation neuromorphic computational infrastructures.

9. Conclusion

This research presented FNGF for uncertain mathematical systems by integrating fuzzy graph theory, adaptive neuromorphic spike propagation, entropy-aware graph optimization, and spectral graph stability analysis within a unified graph intelligence architecture. The proposed framework addressed major limitations of conventional graph learning systems involving uncertainty representation, adaptive graph evolution, graph robustness, energy-efficient computation, and uncertainty propagation optimization.

Experimental evaluations conducted using 5000 synthetic uncertain graph instances demonstrated that the proposed framework significantly outperformed conventional graph learning and uncertainty modeling approaches. The proposed FNGF achieved uncertainty classification accuracies of 97.82%, 94.12%, and 90.37% under low, medium, and high uncertainty environments, respectively. Additionally, the framework demonstrated 95.84% graph stability prediction accuracy and achieved approximately 60.29% graph entropy reduction and nearly 79% graph energy consumption reduction compared to conventional graph neural architectures. The entropy-aware graph pruning mechanism successfully eliminated unstable graph pathways and improved graph sparsification efficiency, while adaptive spike-driven graph communication substantially enhanced graph convergence behavior and reduced computational overhead. Spectral graph stability analysis further improved graph robustness and resilience against topological perturbations and uncertainty propagation.

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CRediT authorship contribution statement

A Sudha: Conceptualization, Investigation, Writing – review & editing. **P Jeyanthi**: Conceptualization & Validation. **Kavitha Chinnathambi**: Data curation & Visualization. **K. Kalaivani**: Data curation & Visualization. **Devika Dabke**: Formal analysis.

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