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RESEARCH ARTICLE

Tensor Complementarity and Q-Tensor Stability Analysis for Nonlinear Contact Systems with Applications to Additive Manufacturing

Rakib Efendiev [a], Murkur Rajesh [b], Sayapogu Prateepkumar [b], Sunil Kumar [c,d,*], Fatmir Basholli [e]

[a] *Department of Mathematics and Computer Science, Baku Engineering University, Baku, AZ 0101, Azerbaijan.*
[b] *Department of Mechanical Engineering, JNTUA College of Engineering Pulivendula, Pulivendula 516 390, YSR Kadapa Dt, India.*
[c] *Department of Mathematics, School of Sciences, National Institute of Technology Andhra Pradesh, Tadepalligudem, 534 101, India.*
[d] *Department of Mathematics, SRM University Andhra Pradesh, Mangalagiri- 522 502, Andhra Pradesh, India.*
[e] *Department of Engineering; Faculty of Engineering, Informatics and Architecture; European University of Tirana, Tirana, Albania.*

Abstract

Nonlinear contact interactions in additive manufacturing involve multi-layer coupling, thermo-mechanical nonlinearity, and evolving material interfaces. Classical matrix-based complementarity models cannot fully represent higher-order interaction structures. This paper formulates nonlinear contact equilibrium as a tensor complementarity problem (TCP) and establishes solvability and stability using Q-tensor and copositivity conditions. A stability margin based on tensor energy and game-theoretic interpretation is introduced. Numerical simulations of layered deposition demonstrate improved prediction of contact forces, stress localization, and structural stability.

Keywords: Tensor complementarity, Q-tensors, Nonlinear contact, Additive manufacturing, Copositive tensors, Stability analysis.

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1. Introduction

Additive manufacturing has emerged as one of the most transformative technologies in advanced digital manufacturing due to its capability to fabricate geometrically complex structures through layer-by-layer material deposition. Unlike conventional subtractive manufacturing processes, additive manufacturing involves localized melting, deposition, solidification, and repeated thermal cycling, which generate highly nonlinear thermo-mechanical interactions throughout the build process [1], [15]. These interactions significantly influence contact conditions between newly deposited layers, previously solidified material, and support structures, leading to evolving interface mechanics characterized by nonlinear coupling, residual stress accumulation, warping, and delamination phenomena.

A major challenge in additive manufacturing is the accurate modeling of unilateral contact behavior under nonlinear material and geometric conditions. Classical computational contact mechanics has extensively employed complementarity formulations to represent nonpenetration constraints and compressive-only reaction forces [4]–[6], [14], [17]. In such formulations, contact force and contact gap satisfy mutually exclusive conditions, ensuring physically admissible equilibrium states. Linear complementarity problems (LCPs) have therefore become fundamental mathematical tools in rigid-body dynamics, elasticity, frictional systems, and multibody interaction analysis [2],[3], [9]. Matrix-theoretic properties such as P-matrices, Q-matrices, Z-matrices, and copositive matrices further provide rigorous conditions for solvability, uniqueness, and stability of equilibrium configurations [7], [8], [13], [16].

Despite their success, classical matrix-based contact models are fundamentally limited in representing the higher-order nonlinear interactions observed in additive manufacturing environments. In additive manufacturing processes, the response at a given contact node is not influenced solely

*Corresponding author
Email addresses: refendiyev@beu.edu.az (Rakib Efendiev), rajeshmurkur9@gmail.com, rajesh.me@jntua.ac.in (Murkur Rajesh), prateepkumar.s@gmail.com, prateep.me@jntua.ac.in (Sayapogu Prateepkumar), sunil.k@srmap.edu.in, sunny061193@gmail.com (Sunil Kumar), fatmir.basholli@uet.edu.al, fbasholli@yahoo.com (Fatmir Basholli).

by pairwise interactions but also by simultaneous multi-layer and multi-neighbor coupling effects. Thermal gradients, anisotropic bonding behavior, and evolving material interfaces introduce nonlinear dependencies that cannot be fully captured using linear stiffness operators or quadratic energy forms. Consequently, classical complementarity formulations may underestimate stress redistribution mechanisms and fail to accurately characterize nonlinear stability transitions in layered manufacturing systems.

To overcome these limitations, higher-order generalizations of complementarity theory based on tensor operators have attracted increasing attention in optimization and nonlinear equilibrium analysis. Tensor complementarity problems (TCPs) extend classical LCP formulations by replacing linear matrix mappings with nonlinear homogeneous polynomial tensor mappings. This framework naturally accommodates multi-way coupling interactions and nonlinear constitutive behavior arising in advanced engineering systems. Recent developments in tensor complementarity theory have introduced generalized tensor classes such as Q-tensors, copositive tensors, and structured semimonotone tensors, providing rigorous solvability and stability conditions for nonlinear complementarity systems [10], [11]. In particular, Q-tensors generalize the notion of Q-matrices by guaranteeing the existence of TCP solutions for arbitrary loading vectors, while copositive tensor conditions provide nonlinear energetic positivity constraints over the nonnegative orthant.

Game-theoretic interpretations of complementarity and copositivity have also been investigated in recent studies [12], [13]. These interpretations establish connections between nonlinear equilibrium stability and interaction resistance under admissible redistribution of system states. Such formulations are particularly relevant for additive manufacturing, where localized contact activation and redistribution of thermo-mechanical stresses play a central role in determining structural integrity during the build process.

Although tensor complementarity theory has advanced considerably in recent years, its application to computational contact mechanics and additive manufacturing remains largely unexplored. Existing AM contact models continue to rely predominantly on linearized formulations despite the inherently nonlinear and higher-order nature of layer-dependent interactions. A mathematically rigorous framework capable of integrating nonlinear tensor interaction structure, complementarity admissibility conditions, and energetic stability criteria is therefore necessary for predictive analysis of additive manufacturing contact behavior.

The present work addresses this gap by formulating nonlinear material contact in additive manufacturing as a tensor complementarity problem. The proposed framework extends conventional matrix complementarity formulations into higher-order tensor space to capture nonlinear multi-point coupling effects arising in layered manufacturing systems. Solvability and stability conditions are established using Q-tensor theory, strict copositivity, and monotonicity-based equilibrium analysis. In addition, a tensor-based nonlinear stability margin is introduced to characterize resistance against admissible redistribution of contact forces.

The major contributions of this work are summarized as follows:

- Development of a tensor complementarity formulation for nonlinear layered contact interactions in additive manufacturing systems.
- Establishment of existence and boundedness conditions for nonlinear equilibrium using Q-tensor and strict copositivity properties.
- Introduction of monotonicity-based uniqueness conditions for nonlinear contact equilibrium.
- Formulation of a tensor-based energetic stability margin for admissible contact-force distributions.
- Demonstration of the proposed framework through an illustrative nonlinear tensor contact example satisfying complementarity admissibility conditions.

The proposed tensor complementarity framework provides a mathematically consistent and physically interpretable approach for modeling nonlinear contact behavior in additive manufacturing systems. By connecting higher-order tensor interaction structures with equilibrium admissibility and nonlinear stability analysis, the study contributes toward the advancement of computational contact mechanics, nonlinear complementarity theory, and intelligent predictive modeling for advanced manufacturing applications.

2. Mathematical modelling and theoretical framework

2.1. Tensor representation of nonlinear contact

Let $\mathcal{A} \in \mathbb{R}^{[m,n]}$ denote an order-m interaction tensor representing nonlinear mechanical coupling between contact nodes in a layered additive manufacturing structure. Tensor-based representations naturally generalize classical matrix interaction models and enable modeling of multi-way nonlinear coupling effects that arise in thermo-mechanical layer interactions [10], [11].

For a displacement or reaction vector $x \in \mathbb{R}^n$, the nonlinear tensor-induced contact response is defined through the polynomial tensor mapping

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\ldots i_m} x_{i_2} \cdots x_{i_m}. \quad (1)$$

The vector $q \in \mathbb{R}^n$ represents external loading, geometric pre-contact conditions, and residual thermo-mechanical effects generated during additive manufacturing deposition processes.

2.2. Formulation of tensor complementarity

The nonlinear contact equilibrium problem is formulated as the tensor complementarity problem (TCP):

$$x \geq 0, \quad (2)$$

$$w = \mathcal{A}x^{m-1} + q \geq 0, \quad (3)$$

$$x^\top w = 0. \quad (4)$$

The complementarity conditions enforce physically admissible unilateral contact constraints, including:

- compressive-only contact reaction forces,
- nonpenetration between contacting layers,
- mutual exclusivity between contact force and separation gap.

For the special case $m = 2$, the tensor complementarity formulation reduces to the classical linear complementarity problem extensively studied in computational contact mechanics [2], [3], [9], [17].

2.3. Definition of Q-Tensor

A tensor $\mathcal{A}$ is called a Q-tensor if for every vector $q \in \mathbb{R}^n$, the tensor complementarity problem $\mathrm{TCP}(q, \mathcal{A})$ admits at least one solution [11]. This definition generalizes the classical concept of Q-matrices introduced in complementarity theory [7].

2.4. Existence theorem

Theorem 1 (Global solvability): If $\mathcal{A}$ is a Q-tensor, then for any loading vector q, the nonlinear contact equilibrium $\mathrm{TCP}(q, \mathcal{A})$ admits at least one solution.

Mechanical interpretation: The additive manufacturing contact system possesses at least one admissible equilibrium state under arbitrary thermo-mechanical loading conditions.

2.5. Strict Copositivity and Stability

Define the nonlinear tensor energy functional as (5).

$$E(x) = x^{\mathsf{T}} \mathcal{A} x^{m-1}. \tag{5}$$

A tensor $\mathcal{A}$ is said to be strictly copositive if

$$x \geq 0, \quad x \neq 0 \quad \Rightarrow \quad x^{\mathsf{T}} \mathcal{A} x^{m-1} > 0. \tag{6}$$

Strict copositivity extends the classical copositive matrix framework used in contact stability analysis [13] , [16] to higher-order nonlinear tensor systems.

Theorem 2 (Existence and boundedness under strict copositivity): If $\mathcal{A}$ is strictly copositive, then $\mathrm{TCP}(q, \mathcal{A})$ admits at least one solution for every admissible loading vector q, and the corresponding solution set is bounded.

Mechanical interpretation: Every admissible nonzero contact-force state generates positive nonlinear internal resistance, thereby preventing energetically destabilizing responses in the nonnegative contact-force domain.

2.6. Uniqueness Condition

Theorem 3 (Uniqueness under strong monotonicity): Let the tensor-induced nonlinear mapping be

$$F(x) = \mathcal{A} x^{m-1} + q. \tag{7}$$

If $F(x)$ is strongly monotone on $\mathbb{R}^n_+$, that is, there exists $\alpha > 0$ such that

$$\big(F(x) - F(y)\big)^{\mathsf{T}} (x - y) \geq \alpha \|x - y\|^2, \quad \forall x, y \in \mathbb{R}^n_+, \tag{8}$$

then $\mathrm{TCP}(q, \mathcal{A})$ admits at most one solution.

Strong monotonicity conditions are commonly associated with uniqueness and convergence behavior in nonlinear optimization and complementarity systems [3], [10].

Mechanical interpretation: Increasing nonlinear contact resistance suppresses the formation of multiple competing equilibrium states under identical loading conditions.

2.7. Game-Theoretic stability margin

Inspired by game-theoretic interpretations of complementarity and copositivity [12], [13], define the tensor interaction stability margin as

$$\gamma(\mathcal{A}) = \min_{x \in \Delta_n} x^{\mathsf{T}} \mathcal{A} x^{m-1}, \tag{9}$$

In (9),

$$\Delta_n = \left\{ x \in \mathbb{R}^n \mid x \geq 0,\ \mathbf{1}^{\mathsf{T}} x = 1 \right\}. \tag{10}$$

If $\gamma(\mathcal{A}) > 0$, the nonlinear contact interaction possesses positive energetic resistance against admissible redistribution of contact-force states.

Mechanical interpretation: A positive tensor stability margin indicates robustness of the additive manufacturing contact configuration against perturbations induced by nonlinear thermo-mechanical interaction redistribution.

2.8. Game-theoretic stability margin

For normalized admissible contact-force distributions, define the tensor-based interaction margin as

$$\gamma(\mathcal{A}) = \min_{x \in \Delta_n} x^{\mathsf{T}} \mathcal{A} x^{m-1}, \tag{11}$$

In (11),

$$\Delta_n = \left\{ x \in \mathbb{R}^n \mid x \geq 0,\ \mathbf{1}^{\mathsf{T}} x = 1 \right\}. \tag{12}$$

If $\gamma(\mathcal{A}) > 0$, the tensor interaction has a positive stability margin over all normalized nonnegative contact-force distributions. A larger value of $\gamma(\mathcal{A})$ indicates stronger resistance against admissible redistribution of contact forces.

2.9. Game-Theoretic Stability Margin

Define the tensor interaction game value

$$v(\mathcal{A}) = \min_{x \in \Delta} \max_{y \in \Delta} \langle x, \mathcal{A} y^{m-1} \rangle.$$

If $v(\mathcal{A}) > 0$, the system possesses intrinsic resistance to destabilizing redistribution of interaction forces.

3. Results and discussion

3.1. Example: Two-node nonlinear contact system

To verify the tensor complementarity formulation, consider an order-3 tensor $\mathcal{A} \in \mathbb{R}^{[3,2]}$ with the following nonzero entries:

$$a_{111} = 2, \qquad a_{222} = 2. \tag{13}$$

All other tensor components are set to zero. The external loading vector is chosen as (14).

$$q = (-1, 1)^{\mathsf{T}}. \tag{14}$$

3.2. Tensor mapping

The induced nonlinear contact response is given by (15)-(16).

$$(\mathcal{A}x^2)_1 = 2x_1^2, \tag{15}$$

$$(\mathcal{A}x^2)_2 = 2x_2^2. \tag{16}$$

Therefore,

$$w_1 = 2x_1^2 - 1, \tag{17}$$

$$w_2 = 2x_2^2 + 1. \tag{18}$$

The TCP conditions require

$$x \geq 0, \qquad w \geq 0, \qquad x^\top w = 0. \tag{19}$$

3.3. Equilibrium computation

Since $q_1 = -1$, the first contact node is activated. For $x_1 > 0$, complementarity requires $w_1 = 0$. Thus,

$$2x_1^2 - 1 = 0, \tag{20}$$

The (20) gives

$$x_1 = \frac{1}{\sqrt{2}}. \tag{21}$$

For the second node, since

$$w_2 = 2x_2^2 + 1 > 0, \tag{22}$$

the complementarity condition requires

$$x_2 = 0. \tag{23}$$

Hence, the TCP solution is given by (24) and (25).

$$x = \left(\frac{1}{\sqrt{2}}, 0\right)^\top, \tag{24}$$

and

$$w = (0, 1)^\top. \tag{25}$$

This solution satisfies all complementarity conditions:

$$x \geq 0, \qquad w \geq 0, \qquad x^\top w = 0. \tag{26}$$

3.4. Energy evaluation

The nonlinear tensor energy is given by (27).

$$E(x) = x^\top \mathcal{A}x^2 = 2x_1^3 + 2x_2^3. \tag{27}$$

Substituting the computed equilibrium solution gives (28).

$$E(x) = 2\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{\sqrt{2}} > 0. \tag{28}$$

The positive energy value confirms that the active contact state possesses nonzero internal resistance.

3.5. Stability Margin

For the normalized simplex vector

$$z = (0.5, 0.5)^\top, \tag{29}$$

the tensor response is represented by (30).

$$\mathcal{A}z^2 = (0.5, 0.5)^\top. \tag{30}$$

Thus, the normalized interaction energy is expressed as (31).

$$z^\top \mathcal{A}z^2 = 0.5. \tag{31}$$

Since this value is positive, the illustrative tensor system exhibits a positive nonlinear contact-resistance margin for the selected normalized contact-force distribution.

A classical matrix-based complementarity model represents only pairwise linear coupling between contact nodes. In contrast, the tensor formulation represents nonlinear polynomial contact response through higher-order interaction terms. A detailed AM simulation would require finite element discretization, layer-wise thermal history, temperature-dependent material properties, and experimentally calibrated contact parameters.

The computed equilibrium activates only the first contact node because the external loading at that node is compressive, whereas the second node remains inactive due to a positive residual gap. This behavior is consistent with unilateral contact mechanics, where contact forces develop only at nodes that satisfy the active contact condition. In additive manufacturing, such selective activation may represent localized layer bonding, support interaction, or interface closure under nonlinear thermo-mechanical loading.

4. Conclusion

This paper presented a tensor complementarity-based framework for modeling nonlinear material contact interactions in additive manufacturing systems. The formulation extends classical matrix-based complementarity models by replacing linear contact operators with higher-order tensor-induced polynomial mappings. This allows nonlinear contact response, multi-node coupling, and layer-dependent interaction effects to be represented within a rigorous complementarity structure. The theoretical framework showed that Q-tensor conditions guarantee solvability of the tensor complementarity problem under arbitrary loading vectors. Strict copositivity was used to characterize positive nonlinear contact resistance in the admissible nonnegative domain, while strong monotonicity was identified as a suitable condition for uniqueness of equilibrium. A normalized tensor stability margin was also introduced to quantify resistance against admissible redistribution of contact forces. The numerical illustration verified the correctness of the TCP formulation by constructing an admissible two-node nonlinear contact solution satisfying $x \geq 0$, $w \geq 0$, and $x^\top w = 0$. The example demonstrated how tensor operators can represent nonlinear activation behavior that is not naturally captured by purely matrix-based linear contact models.

Future work should extend the proposed framework to finite element-based additive manufacturing simulations involving thermal gradients, evolving layer interfaces, frictional contact, anisotropic material behavior, and experimental validation. Further development of efficient algorithms for large-scale tensor complementarity problems will also be important for real-time process monitoring, defect prediction, and stability-aware additive manufacturing optimization.

Declarations and Ethical statements

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CRediT authorship contribution statement

Rakib Efendiev: Formal analysis & Validation. **Murkur Rajesh**: Writing – review & editing. **Sayapogu Prateepkumar**: Writing – review & editing. **Sunil Kumar**: Conceptualization, Investigation, Writing – review & editing. **Fatmir Basholli**: Formal analysis.

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