

RESEARCH ARTICLE**Structural Analysis of Ladder Corona Graphs via Intuitionistic Fuzzy Cube Difference Labeling**S. Jackson ^{a,*}, M. Abirami^b^a*P.G and Research, Department of Mathematics, V.O. Chidambaram College, Thoothukudi-628008, India.*^b*Research Department of Mathematics, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, India.***Abstract**

Intuitionistic fuzzy graph labeling is a useful approach for describing uncertainty based on membership (MS) and non-membership (NMS) values. This work studies Intuitionistic Fuzzy Cube Difference Labeling (IFCDL) on ladder-based graphs and corona products. Explicit labeling techniques are designed for ladder graphs, open ladder graphs, slanting ladder graphs, circular ladder graphs, and Mobius ladder graphs. It is demonstrated that these graphs permit Intuitionistic Fuzzy Cube Difference Graphs (IFCDGs) by assigning distinct MS and NMS values within the range of $[0, 1]$. The suggested framework is adaptable to different graph products and complicated network models, with potential applications in decision-making, communication networks, and uncertainty-based systems.

Keywords: Membership and non-membership values, Intuitionistic Fuzzy Cube Difference Labeling (IFCDL), Intuitionistic Fuzzy Cube Difference Graphs (IFCDGs).

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This is an open-access article distributed under the terms of the [Creative Commons Attribution-NonCommercial 4.0 International License \(CC BY-NC 4.0\)](https://creativecommons.org/licenses/by-nc/4.0/).**1. Introduction**

Graph labeling plays an important role in graph theory with applications in communication networks, coding theory, and network design. To handle uncertainty in real-world systems, fuzzy graph theory extends classical graph theory. The concept of intuitionistic fuzzy sets introduced by Krassimir T. Atanassov incorporates both membership and non-membership degrees, providing a flexible framework for modeling uncertainty [1], [2]. Intuitionistic fuzzy graphs and their structural properties have been widely investigated in the literature [3], [4]. Intuitionistic fuzzy labeling assigns suitable membership and non-membership values to vertices and edges under certain conditions [8]. Cube difference labeling is a structured labeling technique with strong arithmetic properties. Its extension to the intuitionistic fuzzy environment leads to IFCDL [12]- [15]. In this work, we study IFCDL on the corona product of ladder graphs and their variants and prove that these graphs admit intuitionistic fuzzy cube difference graphs.

2. Preliminaries

This section introduces fundamental concepts and definitions essential for the discussion of specialized graph labelings [5]- [7]. A clear understanding of these preliminaries is crucial for the development and analysis of the new labeling techniques presented in this paper [9]- [11].

2.1. Definition

For graphs G and H , the corona product $G \odot H$ is obtained by taking one copy of graph G and $|V(G)|$ copies of graph H and joining each vertex of the i^{th} copy of graph H to the i^{th} vertex of the graph G , where $1 \leq i \leq |V(G)|$.

2.2. Definition

The Cartesian product of P_n and K_2 is called a ladder graph and is denoted as L_n . (i.e.) $L_n = P_n \times K_2$.

2.3. Definition

An open ladder is obtained from a ladder by removing the edges $v_i u_i$ and $v_n u_n$ and is denoted by $O(L_n)$.

*Corresponding author

Email addresses: jmjjack2008@gmail.com (S. Jackson),
abiramimaharajan96@gmail.com (M. Abirami).

2.4. Definition

A slanting ladder SL_n is the graph obtained from two paths of length n say $P_n = (u_1, u_2, u_3, \dots, u_n)$ and $P'_n = (v_1, v_2, v_3, \dots, v_n)$, by joining $u_{(i+1)}$ to v_i for $i = 1$ to $n - 1$ or joining $u_i v_{(i+1)}$ for $i = 1$ to $n - 1$. It is isomorphic to $Z - P_n$.

2.5. Definition

A circular ladder graph denoted by CL_n is obtained by joining parallel vertices of two cycles of the same length.

2.6. Definition

A Mobius ladder graph ML_n is a graph obtained from the ladder graph L_n by adding the edges $u_1 v_n$ and $v_1 u_n$. We denote it as ML_n for clarity, though it is earlier denoted as M_n .

2.7. Definition

If $\sigma_{(\alpha_1)} : V_X \rightarrow [0, 1]$, $\sigma_{(\alpha_2)} : V_X \rightarrow [0, 1]$, and $\mu_{(\alpha_1)} : V_X \times V_X \rightarrow [0, 1]$, $\mu_{(\alpha_2)} : V_X \times V_X \rightarrow [0, 1]$, then an IFG $G = (V_X, \sigma_\alpha, \mu_\alpha)$ is considered an intuitionistic fuzzy labeling graph (IFLG) if it is bijective. In the sense that the vertices and edges have different MS and NMS values, and $\mu_{(\alpha_1)}(v_i, v_j) < \min\{\sigma_{(\alpha_1)}(v_i), \sigma_{(\alpha_1)}(v_j)\}$, $\mu_{(\alpha_1)}(v_i, v_j) < \max\{\sigma_{(\alpha_2)}(v_i), \sigma_{(\alpha_2)}(v_j)\}$, $0 \leq \mu_{(\alpha_1)}(v_i, v_j) + \mu_{(\alpha_2)}(v_i, v_j) \leq 1$ for each edge (v_i, v_j) .

3. Result and discussions

3.1. Definition

An IFG $G = (V, \sigma_\alpha, \mu_\alpha)$ is said to be an Intuitionistic Fuzzy Cube Difference Labeling Graph (IFCDLG) if it is an Intuitionistic Fuzzy Labeling Graph (IFLG) and satisfies the following conditions,

$\mu_{(\alpha_1)}(u_s, v_t) = |\sigma_{(\alpha_1)}(u_s)^3 - \sigma_{(\alpha_1)}(v_t)^3|$, $\mu_{(\alpha_2)}(u_s, v_t) = |\sigma_{(\alpha_2)}(u_s)^3 - \sigma_{(\alpha_2)}(v_t)^3|$, $0 \leq \mu_{(\alpha_1)}(u_s, v_t) + \mu_{(\alpha_2)}(u_s, v_t) \leq 1$ for each $(u_s, v_t) \in E$ are all distinct.

The term IFCDLG refers to a graph that is an IFCDL.

3.2. Theorem

$L_n \odot K_1$ admits an IFCDG.

Proof: Let $G = L_n \odot K_1$ be the $4n$ vertex provided graph. By adding a new vertex to each vertex of L_n , the graph G is created.

Let $\{v_i : 1 \leq i \leq n\}$ be the vertices on one side of L_n and $\{u_i : 1 \leq i \leq n\}$ be the vertices on the other side of L_n .

Let $a^1, a^2, \dots, a^n$ be the n new vertices attached to the corresponding n vertices v_i ($1 \leq i \leq n$), and $b^1, b^2, \dots, b^n$ be the n new vertices attached to the corresponding n vertices u_i ($1 \leq i \leq n$), respectively.

Examine the edge functions μ_1 and μ_2 from each pair of vertices in V to an MS and NMS value between 0 and 1, as well as the vertex functions σ_1 and σ_2 from the vertex V to an MS and NMS value between 0 and 1.

Choose the MS value $\omega_1 \rightarrow (0, 1]$ such that $\omega_1 = 10^{-2}$ for $n \leq 99$, $\omega_1 = 10^{-4}$ for $100 \leq n \leq 999$, $\omega_1 = 10^{-6}$ for $1000 \leq n \leq 9999$ and so on.

Choose the NMS value $\omega_2 \rightarrow (0, 1]$ such that $\omega_2 = 10^{-3}$ for $n \leq 999$, $\omega_2 = 10^{-5}$ for $1000 \leq n \leq 9999$, $\omega_2 = 10^{-7}$ for $10000 \leq n \leq 999999$ and so on.

The labels assigned to the MS value of vertices and edges are,

$$\begin{aligned} \sigma_1(v_i) &= i * \omega_1, & 1 \leq i \leq n \\ \sigma_1(u_i) &= (n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(a^i) &= (2n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(b^i) &= (3n + i) * \omega_1, & 1 \leq i \leq n \\ \mu_1(v_1 v_2) &= 7 * \omega_1^3 \\ \mu_1(v_i v_{i+1}) &= (6 * \omega_1^3 i) + \mu_1(v_{i-1} v_i), & 2 \leq i \leq n - 1 \\ \mu_1(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_1^3 \\ \mu_1(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_1^3 i) + \mu_1(u_{i-1} u_i), & n + 2 \leq i \leq n - 1 \\ \mu_1(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_1^3, & 1 \leq i \leq n \\ \mu_1(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_1^3, & 1 \leq i \leq n \\ \mu_1(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_1^3, & 1 \leq i \leq n \end{aligned}$$

The labels assigned to the NMS value of vertices and edges are,

$$\begin{aligned} \sigma_2(v_i) &= i * \omega_2, & 1 \leq i \leq n \\ \sigma_2(u_i) &= (n + i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(a^i) &= (2n + i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(b^i) &= (3n + i) * \omega_2, & 1 \leq i \leq n \\ \mu_2(v_1 v_2) &= 7 * \omega_2^3 \\ \mu_2(v_i v_{i+1}) &= (6 * \omega_2^3 i) + \mu_2(v_{i-1} v_i), & 2 \leq i \leq n - 1 \\ \mu_2(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_2^3 \\ \mu_2(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_2^3 i) + \mu_2(u_{i-1} u_i), & n + 2 \leq i \leq n - 1 \\ \mu_2(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_2^3, & 1 \leq i \leq n \\ \mu_2(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_2^3, & 1 \leq i \leq n \\ \mu_2(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_2^3, & 1 \leq i \leq n \end{aligned}$$

The MS and NMS values of the vertices and edges are distinct. This labeling obeys the conditions of IFCDL. Thus, $L_n \odot K_1$ is an IFCDG.

3.3. Example

The following Figure 1 illustrates the IFCDG of $L_5 \odot K_1$.

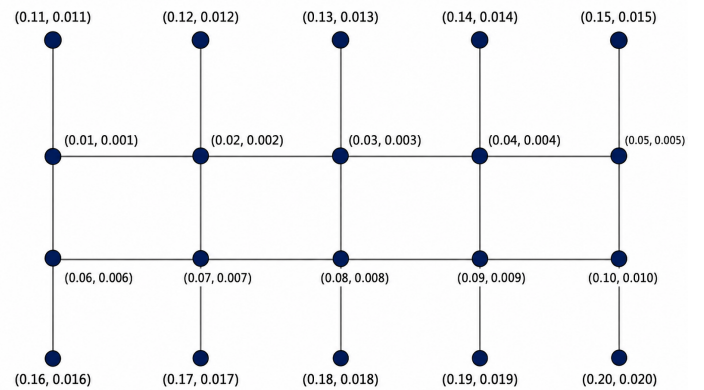


Figure 1: IFCDG for $L_5 \odot K_1$

3.4. Theorem

$OL_n \odot K_1$ admits an intuitionistic fuzzy cube difference graph.

Proof: Let $G = OL_n \odot K_1$ be the corona of an open ladder graph. Let $\{v_i : 1 \leq i \leq n\}$ be the vertices on one side of OL_n and $\{u_i : 1 \leq i \leq n\}$ be the vertices on the other side of OL_n . The G is obtained by attaching one new vertex to every open ladder graph OL_n .

Let $a^1, a^2, \dots, a^n$ be the n new vertices attached to the corresponding n vertices v_i ($1 \leq i \leq n$) and $b^1, b^2, \dots, b^n$ be the n new vertices attached to the corresponding n vertices u_i ($1 \leq i \leq n$), respectively.

Examine the edge functions μ_1 and μ_2 from each pair of vertices in V to an MS and NMS value between 0 and 1, as well as the vertex functions σ_1 and σ_2 from the vertex V to an MS and NMS value between 0 and 1.

Choose the MS value $\omega_1 \rightarrow (0, 1]$ such that $\omega_1 = 10^{-2}$ for $n \leq 99$, $\omega_1 = 10^{-4}$ for $100 \leq n \leq 999$, $\omega_1 = 10^{-6}$ for $1000 \leq n \leq 9999$ and so on.

Choose the NMS value $\omega_2 \rightarrow (0, 1]$ such that $\omega_2 = 10^{-3}$ for $n \leq 999$, $\omega_2 = 10^{-5}$ for $1000 \leq n \leq 9999$, $\omega_2 = 10^{-7}$ for $10000 \leq n \leq 999999$ and so on.

The labels assigned to the MS value of vertices and edges are,

$$\begin{aligned} \sigma_1(v_i) &= i * \omega_1, & 1 \leq i \leq n \\ \sigma_1(u_i) &= (n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(a^i) &= (2n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(b^i) &= (3n + i) * \omega_1, & 1 \leq i \leq n \\ \mu_1(v_1 v_2) &= 7 * \omega_1^3 \\ \mu_1(v_i v_{i+1}) &= (6 * \omega_1^3 i) + \mu_1(v_{i-1} v_i), & 2 \leq i \leq n - 1 \\ \mu_1(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_1^3 \\ \mu_1(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_1^3 i) + \mu_2(u_{i-1} u_i), & n + 2 \leq i \leq n - 1 \\ \mu_1(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_1^3, & 2 \leq i \leq n - 1 \\ \mu_1(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_1^3, & 1 \leq i \leq n \\ \mu_1(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_1^3, & 1 \leq i \leq n \end{aligned}$$

The labels assigned to the NMS value of vertices and edges are,

$$\begin{aligned} \sigma_2(v_i) &= i * \omega_2, & 1 \leq i \leq n \\ \sigma_2(u_i) &= (n + i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(a^i) &= (2n + i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(b^i) &= (3n + i) * \omega_2, & 1 \leq i \leq n \\ \mu_2(v_1 v_2) &= 7 * \omega_2^3 \\ \mu_2(v_i v_{i+1}) &= (6 * \omega_2^3 i) + \mu_2(v_{i-1} v_i), & 2 \leq i \leq n - 1 \\ \mu_2(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_2^3 \\ \mu_2(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_2^3 i) + \mu_2(u_{i-1} u_i), & n + 2 \leq i \leq n - 1 \\ \mu_2(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_2^3, & 2 \leq i \leq n - 1 \\ \mu_2(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_2^3, & 1 \leq i \leq n \\ \mu_2(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_2^3, & 1 \leq i \leq n \end{aligned}$$

The MS and NMS values of the vertices and edges are distinct. This labeling obeys the conditions of IFCDL. Thus, the $OL_n \odot K_1$ graph is an IFCDG.

3.5. Example

Figure 2 illustrates the IFCDG of $OL_5 \odot K_1$.

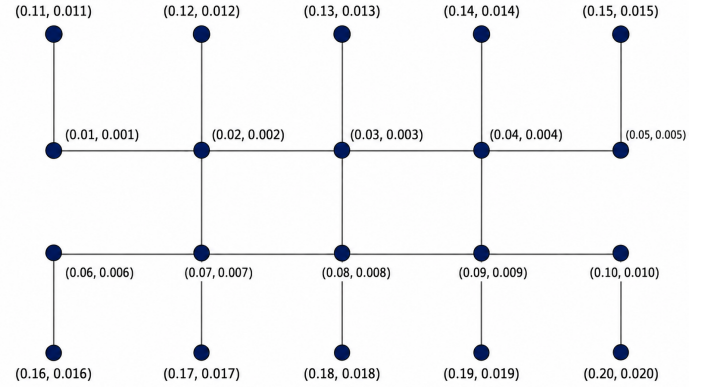


Figure 2: IFCDG for $OL_5 \odot K_1$

3.6. Theorem

$SL_n \odot K_1$ admits an IFCDG.

Proof: Let $G = SL_n \odot K_1$ be the given graph. Let $\{v_i : 1 \leq i \leq n\}$ be the vertices on one side of the slanting ladder and u_i ($1 \leq i \leq n$) be the vertices on the other side of the slanting ladder, and each v_i is adjacent to u_{i+1} for $1 \leq i \leq n - 1$.

The graph G is obtained by attaching one new vertex to every slanting ladder graph SL_n .

Let $a^1, a^2, \dots, a^n$ be the n new vertices attached to the corresponding n vertices v_i ($1 \leq i \leq n$) and $b^1, b^2, \dots, b^n$ be the n new vertices attached to the corresponding n vertices u_i ($1 \leq i \leq n$), respectively.

Examine the edge functions μ_1 and μ_2 from each pair of vertices in V to an MS and NMS value between 0 and 1, as well as the vertex functions σ_1 and σ_2 from the vertex V to an MS and NMS value between 0 and 1.

Choose the MS value $\omega_1 \rightarrow (0, 1]$ such that $\omega_1 = 10^{-2}$ for $n \leq 99$, $\omega_1 = 10^{-4}$ for $100 \leq n \leq 999$, $\omega_1 = 10^{-6}$ for $1000 \leq n \leq 9999$ and so on.

Choose the NMS value $\omega_2 \rightarrow (0, 1]$ such that $\omega_2 = 10^{-3}$ for $n \leq 999$, $\omega_2 = 10^{-5}$ for $1000 \leq n \leq 9999$, $\omega_2 = 10^{-7}$ for $10000 \leq n \leq 999999$ and so on.

The labels assigned to the MS value of vertices and edges are,

$$\begin{aligned} \sigma_1(v_i) &= i * \omega_1, & 1 \leq i \leq n \\ \sigma_1(u_i) &= (n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(a^i) &= (2n + i) * \omega_1, & 1 \leq i \leq n \\ \sigma_1(b^i) &= (3n + i) * \omega_1, & 1 \leq i \leq n \\ \mu_1(v_1 v_2) &= 7 * \omega_1^3 \\ \mu_1(v_i v_{i+1}) &= (6 * \omega_1^3 i) + \mu_1(v_{i-1} v_i), & 2 \leq i \leq n - 1 \\ \mu_1(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_1^3 \\ \mu_1(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_1^3 i) + \mu_2(u_{i-1} u_i), & n + 2 \leq i \leq n - 1 \\ \mu_1(u_i v_{i+1}) &= (n^3 - 1 + 3i[i(n - 1) + (n^2 - 1)]) * \omega_1^3, & 1 \leq i \leq n - 1 \\ \mu_1(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_1^3, & 1 \leq i \leq n \end{aligned}$$

$$\mu_1(v_i a^i) = (8n^3 + 12n^2i + 6ni) * \omega_1^3, \quad 1 \leq i \leq n$$

The labels assigned to the NMS value of vertices and edges are,

$$\begin{aligned} \sigma_2(v_i) &= i * \omega_2, & 1 \leq i \leq n \\ \sigma_2(u_i) &= (n+i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(a^i) &= (2n+i) * \omega_2, & 1 \leq i \leq n \\ \sigma_2(b^i) &= (3n+i) * \omega_2, & 1 \leq i \leq n \\ \mu_2(v_1 v_2) &= 7 * \omega_2^3 \\ \mu_2(v_i v_{i+1}) &= (6 * \omega_2^3 i) + \mu_2(v_{i-1} v_i), & 2 \leq i \leq n-1 \\ \mu_2(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_2^3 \\ \mu_2(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_2^3 i) + \mu_2(u_{i-1} u_i), & n+2 \leq i \leq n-1 \\ \mu_2(u_i v_{i+1}) &= (n^3 - 1 + 3i[i(n-1) + (n^2 - 1)]) * \omega_2^3, & 1 \leq i \leq n-1 \\ \mu_2(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_2^3, & 1 \leq i \leq n \\ \mu_2(v_i a^i) &= (8n^3 + 12n^2i + 6ni) * \omega_2^3, & 1 \leq i \leq n \end{aligned}$$

The MS and NMS values of the vertices and edges are distinct. This labeling obeys the conditions of IFCDL. Thus, the $SL_n \odot K_1$ graph is an IFCDG.

3.7. Example

The following Figure 3 illustrates the IFCDG of $SL_5 \odot K_1$.

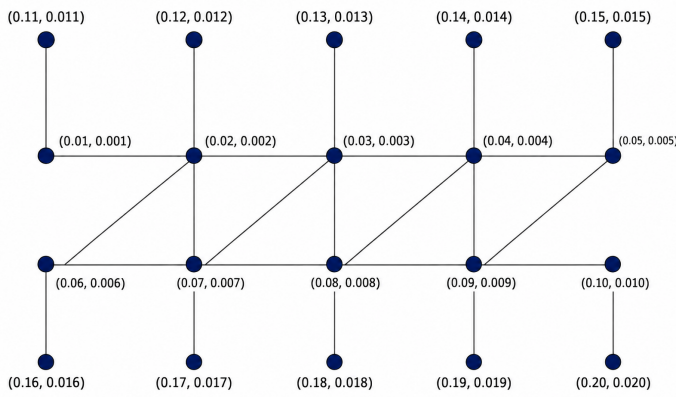


Figure 3: IFCDG for $SL_5 \odot K_1$

3.8. Theorem

$CL_n \odot K_1$ admits an IFCDG.

Proof: Let $G = CL_n \odot K_1$ be a graph.

Let $V(CL_n \odot K_1) = \{v_i, u_i, a^i, b^i : 1 \leq i \leq n\}$ and $E(CL_n \odot K_1) = \{v_i v_{i+1}, u_i u_{i+1}, v_i u_i, v_i a^i, v_i b^i : 1 \leq i \leq n\} \cup \{v_1 v_n, u_1 u_n\}$.

Examine the edge functions μ_1 and μ_2 from each pair of vertices in V to an MS and NMS value between 0 and 1, as well as the vertex functions σ_1 and σ_2 from the vertex V to an MS and NMS value between 0 and 1.

Choose the MS value $\omega_1 \rightarrow (0, 1]$ such that $\omega_1 = 10^{-2}$ for $n \leq 99$, $\omega_1 = 10^{-4}$ for $100 \leq n \leq 999$, $\omega_1 = 10^{-6}$ for $1000 \leq n \leq 9999$ and so on.

Choose the NMS value $\omega_2 \rightarrow (0, 1]$ such that $\omega_2 = 10^{-3}$ for $n \leq 999$, $\omega_2 = 10^{-5}$ for $1000 \leq n \leq 9999$, $\omega_2 = 10^{-7}$

for $10000 \leq n \leq 999999$ and so on.

The labels assigned to the MS value of vertices and edges are,

$$\begin{aligned} \sigma_1(v_i) &= i * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(u_i) &= (n+i) * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(a^i) &= (2n+i) * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(b^i) &= (3n+i) * \omega_1, & 1 \leq i \leq n. \\ \mu_1(v_1 v_2) &= 7 * \omega_1^3, \\ \mu_1(v_i v_{i+1}) &= (6 * \omega_1^3 i) + \mu_1(v_{i-1} v_i), & 2 \leq i \leq n-1, \\ \mu_1(v_1 v_n) &= (n^3 - 1) * \omega_1^3, \\ \mu_1(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_1^3, \\ \mu_1(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_1^3 i) + \mu_1(u_{i-1} u_i), & n+2 \leq i \leq n-1, \\ \mu_1(u_1 u_n) &= (7n^3 - 3n^2 - 3n - 1) * \omega_1^3, \\ \mu_1(u_i v_i) &= n[n^2 + 3i(n+i)] * \omega_1^3, & 1 \leq i \leq n, \\ \mu_1(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_1^3, & 1 \leq i \leq n, \\ \mu_1(v_i a^i) &= (8n^3 + 12n^2i + 6ni) * \omega_1^3, & 1 \leq i \leq n. \end{aligned}$$

The labels assigned to the NMS value of vertices and edges are,

$$\begin{aligned} \sigma_2(v_i) &= i * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(u_i) &= (n+i) * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(a^i) &= (2n+i) * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(b^i) &= (3n+i) * \omega_2, & 1 \leq i \leq n. \\ \mu_2(v_1 v_2) &= 7 * \omega_2^3, \\ \mu_2(v_i v_{i+1}) &= (6 * \omega_2^3 i) + \mu_2(v_{i-1} v_i), & 2 \leq i \leq n-1, \\ \mu_2(v_1 v_n) &= (n^3 - 1) * \omega_2^3, \\ \mu_2(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_2^3, \\ \mu_2(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_2^3 i) + \mu_2(u_{i-1} u_i), & n+2 \leq i \leq n-1, \\ \mu_2(u_1 u_n) &= (7n^3 - 3n^2 - 3n - 1) * \omega_2^3, \\ \mu_2(u_i v_i) &= n[n^2 + 3i(n+i)] * \omega_2^3, & 1 \leq i \leq n, \\ \mu_2(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_2^3, & 1 \leq i \leq n, \\ \mu_2(v_i a^i) &= (8n^3 + 12n^2i + 6ni) * \omega_2^3, & 1 \leq i \leq n. \end{aligned}$$

The MS and NMS values of the vertices and edges are distinct. This labeling obeys the conditions of IFCDL. Thus, the $CL_n \odot K_1$ graph is an IFCDG.

3.9. Example

The following Figure 4 illustrates the IFCDG of $CL_5 \odot K_1$.

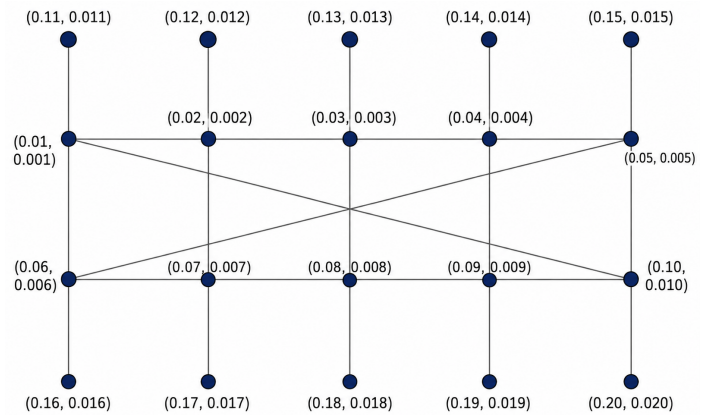


Figure 4: IFCDG for $CL_5 \odot K_1$

3.10. Theorem

$ML_n \odot K_1$ admits an IFCDL.

Proof: Let $G = ML_n \odot K_1$ be a graph.

Let $V(ML_n \odot K_1) = \{v_i, u_i, a^i, b^i : 1 \leq i \leq n\}$ and $E(ML_n \odot K_1) = \{v_i v_{i+1}, u_i u_{i+1}, v_i u_i, v_i a^i, v_i b^i : 1 \leq i \leq n\} \cup \{v_1 u_n, u_1 v_n\}$.

Examine the edge functions μ_1 and μ_2 from each pair of vertices in V to an MS and NMS value between 0 and 1, as well as the vertex functions σ_1 and σ_2 from the vertex V to an MS and NMS value between 0 and 1.

Choose the MS value $\omega_1 \rightarrow (0, 1]$ such that $\omega_1 = 10^{-2}$ for $n \leq 99$, $\omega_1 = 10^{-4}$ for $100 \leq n \leq 999$, $\omega_1 = 10^{-6}$ for $1000 \leq n \leq 9999$ and so on.

Choose the NMS value $\omega_2 \rightarrow (0, 1]$ such that $\omega_2 = 10^{-3}$ for $n \leq 999$, $\omega_2 = 10^{-5}$ for $1000 \leq n \leq 9999$, $\omega_2 = 10^{-7}$ for $10000 \leq n \leq 999999$ and so on.

The labels assigned to the MS value of vertices and edges are,

$$\begin{aligned} \sigma_1(v_i) &= i * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(u_i) &= (n + i) * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(a^i) &= (2n + i) * \omega_1, & 1 \leq i \leq n, \\ \sigma_1(b^i) &= (3n + i) * \omega_1, & 1 \leq i \leq n. \\ \mu_1(v_1 v_2) &= 7 * \omega_1^3, \\ \mu_1(v_i v_{i+1}) &= (6 * \omega_1^3 i) + \mu_1(v_{i-1} v_i), & 2 \leq i \leq n - 1, \\ \mu_1(v_1 u_n) &= [(2n)^3 - 1] * \omega_1^3, \\ \mu_1(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_1^3, \\ \mu_1(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_1^3 i) + \mu_1(u_{i-(n+1)} u_{i-n}), & n + 2 \leq i \leq n - 1, \\ \mu_1(u_1 v_n) &= [3n(n + 1) + 1] * \omega_1^3, \\ \mu_1(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_1^3, & 1 \leq i \leq n, \\ \mu_1(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_1^3, & 1 \leq i \leq n, \\ \mu_1(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_1^3, & 1 \leq i \leq n. \end{aligned}$$

The labels assigned to the NMS value of vertices and edges are,

$$\begin{aligned} \sigma_2(v_i) &= i * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(u_i) &= (n + i) * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(a^i) &= (2n + i) * \omega_2, & 1 \leq i \leq n, \\ \sigma_2(b^i) &= (3n + i) * \omega_2, & 1 \leq i \leq n. \\ \mu_2(v_1 v_2) &= 7 * \omega_2^3, \\ \mu_2(v_i v_{i+1}) &= (6 * \omega_2^3 i) + \mu_2(v_{i-1} v_i), & 2 \leq i \leq n - 1, \\ \mu_2(v_1 u_n) &= [(2n)^3 - 1] * \omega_2^3, \\ \mu_2(u_1 u_2) &= (3n^2 + 9n + 7) * \omega_2^3, \\ \mu_2(u_{i-n} u_{i-(n-1)}) &= (6 * \omega_2^3 i) + \mu_2(u_{i-(n+1)} u_{i-n}), & n + 2 \leq i \leq n - 1, \\ \mu_2(u_1 v_n) &= [3n(n + 1) + 1] * \omega_2^3, \\ \mu_2(u_i v_i) &= n[n^2 + 3i(n + i)] * \omega_2^3, & 1 \leq i \leq n, \\ \mu_2(u_i b^i) &= 2n(13n^2 + 12ni + 3i^2) * \omega_2^3, & 1 \leq i \leq n, \\ \mu_2(v_i a^i) &= (8n^3 + 12n^2 i + 6ni) * \omega_2^3, & 1 \leq i \leq n. \end{aligned}$$

The MS and NMS values of the vertices and edges are distinct. This labeling obeys the conditions of IFCDL. Thus, the $ML_n \odot K_1$ graph is an IFCDG.

3.11. Example

Figure 5 illustrates the IFCDG of $ML_5 \odot K_1$.

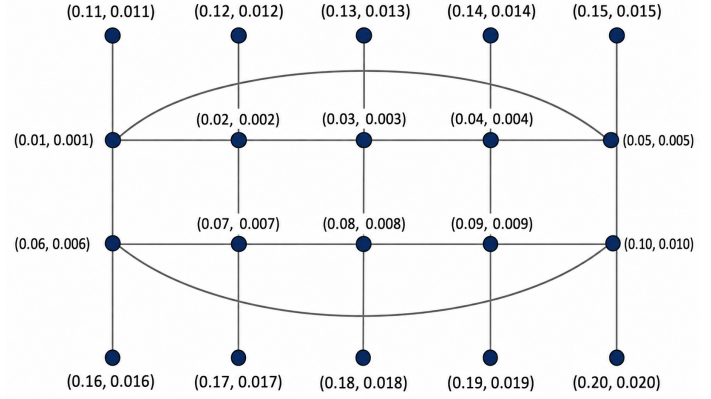


Figure 5: IFCDG for $ML_5 \odot K_1$

4. Conclusion

This paper investigated Intuitionistic Fuzzy Cube Difference Labeling for the corona product of ladder graphs and their variants. Suitable membership and non-membership labeling functions were constructed, and it was shown that these graph classes admit intuitionistic fuzzy cube difference graphs. The results extend existing intuitionistic fuzzy labeling studies to structured corona graph families. The proposed framework can be further extended to other graph products and complex networks. Future work may explore algorithmic implementations and applications in communication systems, decision-making models, and uncertainty-based network analysis.

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CRedit authorship contribution statement

S. Jackson: Conceptualization, Investigation, Visualization, Writing – original draft, review & editing. **M. Abirami:** Conceptualization, Methodology, Writing – original draft, review & editing, Validation.

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