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REVIEW ARTICLE

A Comprehensive Review on Advanced Optimization Techniques for Antenna Design

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Abstract

The rapid evolution of wireless communication systems has transformed antenna design from a predominantly analytical engineering task into a nonlinear, high-dimensional, and multi-objective optimization problem. Emerging applications such as fifth-generation (5G) and sixth-generation (6G) wireless networks, massive multiple-input multiple-output (MIMO) systems, millimeter-wave communication, satellite constellations, vehicular radar, biomedical implants, wearable electronics, and Internet-of-Things platforms impose stringent requirements on antenna compactness, bandwidth, gain, radiation efficiency, polarization stability, beam steering capability, mutual coupling suppression, and fabrication tolerance. This review presents a systematic and analytical examination of optimization techniques used in antenna design, covering deterministic methods, evolutionary algorithms, swarm intelligence, multi-objective optimization, surrogate-assisted optimization, space mapping, robust optimization, and artificial intelligence-driven frameworks. Special emphasis is placed on the role of surrogate modeling, physics-informed learning, hybrid search strategies, and uncertainty-aware optimization in reducing simulation cost and improving design reliability. The review further identifies unresolved challenges related to scalability, interpretability, convergence assurance, manufacturing uncertainty, and real-time adaptive antenna systems. Finally, future research directions are outlined toward physics-guided, data-efficient, robust, and autonomous optimization frameworks for next-generation electromagnetic systems.

Keywords: Fifth-generation (5G), Sixth-generation (6G), Massive multiple-input multiple-output (MIMO) systems, Antenna optimization, Multi-objective optimization, Artificial intelligence, Electromagnetic simulation.

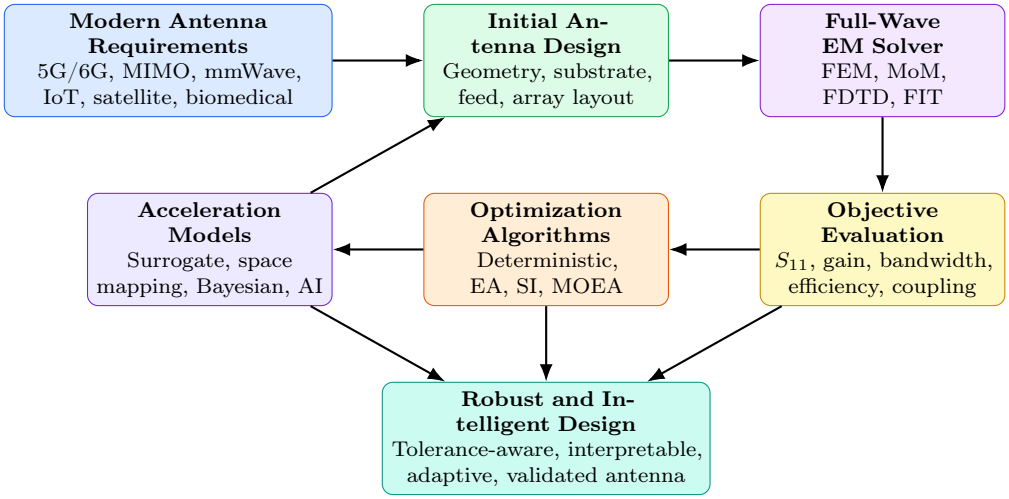
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1. Introduction

The rapid evolution of modern wireless communication systems has significantly transformed antenna design from a predominantly analytical discipline into a complex, simulation-driven and optimization-intensive engineering problem. Emerging technologies such as fifth-generation (5G) and sixth-generation (6G) wireless networks, massive multiple-input multiple-output systems, millimeter-wave communication, satellite constellations, vehicular radar, wearable electronics, biomedical implants, and IoT platforms impose stringent and often conflicting requirements on antenna performance [1], [2], [3], [87], [88]. These requirements include compact size, wide impedance bandwidth, high gain, stable radiation patterns, low mutual coupling, polarization diversity, and robustness under fabrication and environmental uncertainties.

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Graphical Abstract: Optimization-driven antenna design.



Conventional antenna design methodologies are primarily based on electromagnetic theory, including Maxwell's equations, transmission-line models, cavity formulations, and array factor analysis [4], [5]. While these approaches provide fundamental physical insight and initial design approximations, they are often inadequate for modern antenna structures characterized by strong parameter coupling, nonlinear resonance behavior, and complex geometries such as fractal antennas, metamaterial-inspired radiators, dielectric resonator antennas, and reconfigurable systems [6], [7]. As a result, antenna synthesis has increasingly relied on CEM techniques combined with advanced optimization strategies.

The integration of full-wave electromagnetic solvers, including the FEM, MoM, and FDTD, has enabled accurate modeling of complex antenna configurations [8], [9]. However, these solvers introduce substantial computational overhead when used within iterative optimization loops. Since antenna optimization typically involves high-dimensional design spaces and multiple performance objectives evaluated across frequency and spatial domains, direct simulation-based optimization becomes computationally prohibitive. This challenge has driven the adoption of a wide range of optimization techniques, including deterministic methods, evolutionary algorithms, swarm intelligence approaches, multi-objective optimization (MOO) frameworks, surrogate-assisted techniques, and more recently, artificial intelligence-based models.

From a mathematical perspective, antenna design can be formulated as a nonlinear, constrained, and often MOO problem, where the objective functions are implicitly defined through electromagnetic simulations. These problems are typically characterized by multimodal search landscapes, strong variable coupling, and expensive function evaluations. Consequently, no single optimization technique is universally effective across all antenna design scenarios. Instead, different classes of algorithms offer complementary strengths in terms of global exploration, local refinement, computational efficiency, and robustness.

In recent years, surrogate modeling and space-mapping techniques have emerged as powerful tools for reducing the computational burden associated with full-wave simulations. Simultaneously, artificial intelligence and machine learning approaches have introduced new capabilities for data-driven antenna modeling, inverse design, and real-time adaptive optimization [12], [13], [37]. These developments have shifted antenna design toward hybrid frameworks that combine physics-based modeling, numerical optimization, and data-driven intelligence.

Despite significant progress, several challenges remain unresolved. These include scalability for high-dimensional antenna arrays, convergence reliability in multimodal landscapes, interpretability of AI-based models, robustness under manufacturing uncertainties, and the lack of unified frameworks that integrate electromagnetic theory with data-driven optimization. Furthermore, the increasing complexity of next-generation antenna systems necessitates optimization methods that are not only accurate but also computationally efficient and adaptable in real time.

Motivated by these challenges, this paper presents a comprehensive and structured review of optimization techniques for antenna design. The contributions of this review are summarized as follows:

- A unified taxonomy of antenna optimization techniques is presented, covering deterministic, evolutionary, swarm-based, multi-objective, surrogate-assisted, and artificial intelligence-driven methods.
- The mathematical foundations of antenna optimization are systematically formulated, including objective functions, constraints, and Pareto-optimal trade-offs.
- The characteristics of antenna optimization landscapes, including nonlinearity, multimodality, and high dimensionality, are analyzed in detail.
- A critical comparison of optimization algorithms is provided in terms of convergence behavior, computational cost, robustness, and applicability to different antenna classes.
- Emerging trends such as physics-informed learning, digital twin-based optimization, and real-time adaptive antenna systems are discussed.

Mathematical symbols and notations.

Symbol	Description
$f(\mathbf{x})$	Objective function
$\mathbf{F}(\mathbf{x})$	Function vector of MOO
$g_i(\mathbf{x})$	Inequality constraint function
$h_j(\mathbf{x})$	Equality constraint function
S_{21}	Mutual coupling coefficient
G	Antenna gain
BW	Antenna bandwidth
η	Learning rate or optimization step size
$\nabla f(\mathbf{x})$	Gradient of objective function
$\mathbf{H}$	Hessian matrix
$\mathbf{v}_i$	Velocity vector in PSO
$\mathcal{L}_{physics}$	Physics-informed loss component
$\mathbf{p}_i$	Personal best position in PSO
$\mathbf{g}$	Global best position in PSO
w	Inertia weight in PSO
c_1	Cognitive coefficients in PSO
c_2	Social coefficients in PSO
r_1, r_2	Random coefficients in PSO
α	Scaling or weighting coefficient
β	Attraction coefficient in FA
γ	Light absorption coefficient
$\phi(\cdot)$	Radial basis kernel function
$\hat{f}(\mathbf{x})$	Surrogate model approximation
μ	Mean value in Gaussian process
σ^2	Variance parameter
$\mathcal{L}$	Loss function
$\mathbf{E}$	Electric field intensity
$\mathbf{B}$	Magnetic flux density
Z_{in}	Input impedance of antenna
R	Resistance component
X	Reactance component
λ	Regularization coefficient
ϵ	Error/noise term
$\mathcal{M}_\theta$	AI/ML predictive model
R_t	Reinforcement learning reward
$\mathbf{x}^*$	Optimal antenna design solution
$f_i(\mathbf{x})$	Individual objective function
m	Number of objective functions
n	Number of optimization variables
D	Antenna directivity
f_r	Resonant frequency
r_{ij}	Distance between fireflies i and j
δ	Mutation perturbation parameter
$\mathbf{X}_\alpha$	Alpha-wolf position in GWO
$\mathbf{X}_\beta$	Beta-wolf position in GWO
$\mathbf{X}_\delta$	Delta-wolf position in GWO
$\mu(\mathbf{x})$	Mean prediction of Gaussian process
$\sigma^2(\mathbf{x})$	Prediction variance of Gaussian process
$\mathcal{L}_{data}$	Data fitting loss component
θ	Trainable parameters of AI/ML model
$\mathbf{y}_{target}$	Desired antenna performance vector
$Q(s,a)$	State-action value function
$\mathcal{H}$	Hybrid optimization framework
$\mathbf{x}_{GA}$	Solution generated by GA
$\mathbb{R}^n$	n-dimensional continuous search space

List of acronyms.

Acronym	Expansion
5G	Fifth-generation
6G	Sixth-generation
AI	Artificial Intelligence
ANN	Artificial Neural Network
CMA	Covariance Matrix Adaptation
IoT	Internet-of-Things
RL	Reinforcement Learning
DE	Differential Evolution
EA	Evolutionary Algorithm
EM	Electromagnetic
FDTD	Finite Difference Time Domain
FEM	Finite Element Method
FIT	Finite Integration Technique
GA	Genetic Algorithm
DE	Differential Evolution
FA	Firefly Algorithm
GWO	Grey Wolf Optimizer
SQP	Sequential Quadratic Programming
MIMO	Multiple Input Multiple Output
MoM	Method of Moments
MOEA	Multi-Objective Evolutionary Algorithm
NSGA	Non-dominated Sorting Genetic Algorithm
PINN	Physics-Informed Neural Network
VSWR	Voltage Standing Wave Ratio
PSO	Particle Swarm Optimization
ACO	Ant Colony Optimization
MOO	Multi-objective optimization
RBF	Radial Basis Function
RL	Reinforcement Learning
SI	Swarm Intelligence
THz	Terahertz
VAE	Variational Autoencoder
CEM	Computational Electromagnetic
SQP	Sequential Quadratic Programming
PINN	Physics-Informed Neural Networks

2. Historical development of antenna optimization

The evolution of antenna optimization reflects the broader transition of electromagnetic engineering from analytical modeling to computational intelligence-driven design. In the early stages of antenna engineering, design methodologies were primarily based on closed-form analytical solutions derived from Maxwell's equations, transmission-line approximations, and empirical design rules. Classical antenna structures such as dipoles, monopoles, horn antennas, and microstrip patch antennas were synthesized using simplified models that provided acceptable accuracy for relatively simple geometries and narrow-band applications [1], [4]. A simplified microstrip antenna is shown in Figure 1.

However, as communication systems advanced toward higher frequencies, wider bandwidths, and more compact form factors, these analytical approaches became insufficient. The emergence of complex antenna structures-such as multiband antennas, conformal antennas, fractal geometries, and dielectric resonator antennas-introduced strong nonlinear interactions among design parameters. These

complexities necessitated the adoption of numerical methods and computational optimization techniques.

The introduction of CEM methods in the 1970s and 1980s marked a significant milestone in antenna design. Techniques such as the MoM, FEM, and FDTD enabled accurate modeling of arbitrary antenna geometries. However, these methods required iterative parameter tuning, which was initially performed manually by designers. This process was time-consuming and often suboptimal, particularly for high-dimensional design problems.

The 1990s witnessed the integration of optimization algorithms with electromagnetic simulation tools. Genetic algorithms were among the first stochastic optimization techniques widely adopted for antenna design due to their ability to handle nonlinear, multimodal, and discrete design spaces [10], [14]. GA-based methods were successfully applied to antenna array synthesis, Yagi–Uda antenna optimization, and fractal antenna generation. Their population-based search mechanism allowed global exploration of complex design spaces without requiring gradient information.

Following the success of genetic algorithms, differential evolution and evolution strategies further improved optimization performance for continuous antenna parameter tuning [15]. These methods demonstrated better convergence stability and robustness in high-dimensional design spaces compared with earlier evolutionary approaches.

In parallel, swarm intelligence techniques emerged as efficient alternatives to evolutionary algorithms. PSO, introduced in the mid-1990s, gained popularity due to its simple implementation, fast convergence, and reduced number of control parameters [16]. PSO has been extensively applied to antenna array synthesis, beamforming optimization, and impedance matching problems. Other nature-inspired algorithms, such as ant colony optimization, firefly algorithm, and grey wolf optimizer, were subsequently explored for electromagnetic applications, particularly in array thinning and pattern synthesis.

The early 2000s marked the development of multi-objective optimization techniques in antenna design. Traditional single-objective formulations were inadequate for handling conflicting design requirements such as bandwidth versus gain, size versus efficiency, and sidelobe level versus directivity. Multi-objective evolutionary algorithms, including the Non-dominated Sorting Genetic Algorithm II (NSGA-II), enabled the generation of Pareto-optimal solutions, allowing designers to systematically explore trade-offs among competing objectives [17]. This development significantly improved the flexibility and practicality of antenna optimization frameworks.

As computational complexity increased, surrogate-assisted optimization methods were introduced to reduce the number of expensive full-wave electromagnetic simulations. These methods employ approximate models—such as kriging, radial basis function networks, and polynomial response surfaces—to predict antenna performance with significantly lower computational cost [11]. Space mapping techniques further enhanced efficiency by linking coarse analytical models with high-fidelity EM simulations, thereby accelerating convergence while maintaining accuracy.

In recent years, artificial intelligence and machine learning techniques have transformed antenna optimization into a data-driven paradigm. Neural networks, support vector machines, and deep learning models have been used for both forward modeling and inverse antenna design [12], [18]. These approaches enable rapid performance prediction and automated geometry synthesis, significantly reducing design cycle time. Additionally, reinforcement learning has been explored for adaptive antenna tuning and reconfigurable systems, while physics-informed neural networks have been introduced to incorporate electromagnetic constraints directly into learning models.

Overall, the historical development of antenna optimization can be categorized into four major phases:

1. Analytical design phase: Based on closed-form electromagnetic solutions and empirical models.
2. Numerical simulation Phase: Introduction of CEM methods enabling accurate modeling of complex structures.
3. Stochastic optimization phase: Adoption of evolutionary and swarm-based algorithms for global search.
4. Intelligent optimization phase: Integration of surrogate models, machine learning, and hybrid frameworks.

3. Theoretical foundations of antenna optimization

Antenna optimization problems are fundamentally formulated within the framework of nonlinear constrained optimization, where the objective is to determine an optimal set of design variables that satisfies electromagnetic performance requirements while adhering to physical and technological constraints. Unlike conventional optimization problems, antenna design involves objective functions that are implicitly defined through CEM simulations, making the problem highly nonlinear, multimodal, and computationally expensive.

3.1. Design variables and parameter space

Let the antenna design be represented by a vector of decision variables (1):

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \Omega, \tag{1}$$

In (1), $\mathbf{x}$ includes geometric parameters (e.g., patch length, width, slot dimensions), material properties (e.g., dielectric constant, substrate thickness), and excitation parameters (e.g., feed location, phase, amplitude). The feasible design space Ω is defined by fabrication limits, electromagnetic constraints, and application-specific requirements.

The dimensionality of $\mathbf{x}$ can range from a few variables in simple antennas to hundreds or thousands in large antenna arrays and topology optimization problems.

3.2. Single-objective optimization formulation

A general antenna optimization problem can be expressed as (2):

$$\min_{\mathbf{x}\in\Omega} f(\mathbf{x}), \tag{2}$$

subject to (3)-(4):

$$g_i(\mathbf{x}) \le 0, \quad i = 1, 2, \ldots, p, \tag{3}$$

$$h_j(\mathbf{x}) = 0, \quad j = 1, 2, \ldots, q, \tag{4}$$

Typical antenna optimization objectives include (5)-(8):

- Return loss minimization:

$$f_1(\mathbf{x}) = \max_{f \in \mathcal{F}} |S_{11}(\mathbf{x}, f)| \tag{5}$$

- Gain maximization:

$$f_2(\mathbf{x}) = -G(\mathbf{x}) \tag{6}$$

- Bandwidth maximization:

$$f_3(\mathbf{x}) = -BW(\mathbf{x}) \tag{7}$$

- Mutual coupling minimization:

$$f_4(\mathbf{x}) = |S_{21}(\mathbf{x})| \tag{8}$$

Since most EM responses are obtained via numerical solvers, $f(\mathbf{x})$ is generally a black-box function without explicit analytical gradients.

3.3. Constraint modeling in antenna design

Constraints in antenna optimization arise from multiple sources are given in this subsection and these four constraints, such as (19)-(13) ensure that optimized antenna designs remain feasible, manufacturable, and compliant with system requirements.

1. Geometric Constraints:

$$x_i^{\min} \leq x_i \leq x_i^{\max} \tag{9}$$

2. Performance Constraints:

$$|S_{11}(\mathbf{x}, f)| \leq -10 \text{ dB} \tag{10}$$

$$G(\mathbf{x}) \geq G_{\min} \tag{11}$$

3. Physical Constraints:

$$\text{Size}(\mathbf{x}) \leq S_{\max} \tag{12}$$

4. Manufacturing Constraints:

$$\delta x_i \leq \epsilon_i \tag{13}$$

3.4. Multi-objective optimization framework

In practical antenna design, multiple conflicting objectives must be optimized simultaneously. The problem is therefore formulated as (14):

$$\min_{\mathbf{x} \in \Omega} \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T \tag{14}$$

In (14), each $f_i(\mathbf{x})$ represents a performance metric such as gain, bandwidth, efficiency, or size.

A solution $\mathbf{x}_a$ is said to dominate another solution $\mathbf{x}_b$ if:

$$f_i(\mathbf{x}_a) \leq f_i(\mathbf{x}_b) \quad \forall i \tag{15}$$

and

$$f_j(\mathbf{x}_a) < f_j(\mathbf{x}_b) \quad \text{for at least one } j. \tag{16}$$

The set of non-dominated solutions forms the Pareto-optimal front [17], [19], [20]. This front represents trade-offs among conflicting objectives, allowing designers to select solutions based on application priorities.

3.5. Objective aggregation techniques

In some cases, multi-objective problems are converted into single-objective problems using aggregation methods (17)-(18):

- Weighted sum method:

$$f(\mathbf{x}) = \sum_{i=1}^{m} w_i f_i(\mathbf{x}) \tag{17}$$

- ϵ-constraint method:

$$\min f_1(\mathbf{x}) \quad \text{subject to } f_i(\mathbf{x}) \leq \epsilon_i \tag{18}$$

However, these methods may fail to capture non-convex Pareto fronts, making evolutionary multi-objective algorithms more suitable [19].

3.6. Sensitivity analysis and gradient computation

Gradient-based optimization requires computation of sensitivities as given by (19).

$$\nabla f(\mathbf{x}) = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right]^T \tag{19}$$

In electromagnetic problems, sensitivities can be computed by using finite difference approximation, adjoint sensitivity methods and automatic differentiation.

Adjoint methods are particularly efficient because they require only a small number of EM simulations regardless of the number of design variables [11], [50].

3.7. Uncertainty-aware optimization formulation

Considering fabrication tolerances and environmental variability, antenna optimization can be extended to a stochastic framework is expressed by (20) and robust formulation includes variance minimization by (21).

$$\min_{\mathbf{x} \in \Omega} \mathbb{E}_\delta[f(\mathbf{x}, \delta)] \tag{20}$$

In (20), δ represents uncertain parameters.

$$\min \left(\mathbb{E}[f(\mathbf{x}, \delta)] + \lambda \, \text{Var}[f(\mathbf{x}, \delta)] \right) \tag{21}$$

3.8. Computational complexity considerations

The computational cost of antenna optimization is dominated by electromagnetic simulations. If N_{eval} represents the number of function evaluations and T_{EM} represents the simulation time, the total cost is given by (22), in which reducing N_{eval} through surrogate modeling or efficient optimization strategies is therefore critical for practical applications [30], [31].

$$T_{total} = N_{eval} \times T_{EM} \tag{22}$$

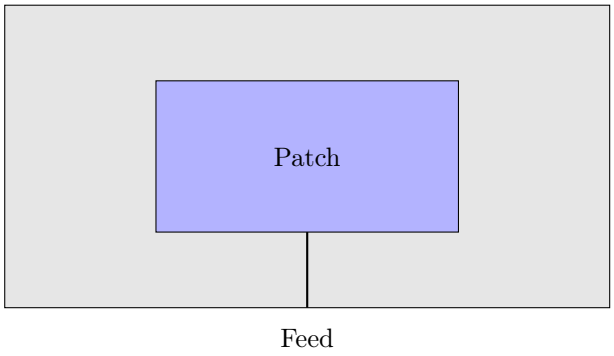


Figure 1: Microstrip patch antenna structure.

3.9. Summary of section

The theoretical formulation of antenna optimization highlights several key challenges:

- Nonlinearity and multimodality of objective functions
- High-dimensional design spaces
- Expensive black-box evaluations
- Conflicting multi-objective requirements
- Uncertainty in fabrication and operation

These challenges motivate the development of advanced optimization algorithms, hybrid frameworks, and data-driven approaches, which are discussed in subsequent sections [48], [54].

4. Optimization landscape analysis

The effectiveness of any optimization algorithm in antenna design is fundamentally influenced by the characteristics of the underlying objective function landscape. Unlike standard convex optimization problems, antenna optimization involves highly nonlinear, multimodal, and computationally expensive response surfaces derived from full-wave electromagnetic simulations [61], [62]. Understanding the structure of this landscape is therefore critical for selecting appropriate optimization strategies [82]–[84].

Electromagnetic optimization problems are particularly challenging because antenna performance metrics depend on coupled physical phenomena governed by Maxwell's equations. Consequently, the resulting objective functions often exhibit multiple local optima, strong parameter interactions, discontinuities, and uncertainty arising from numerical approximations. These characteristics significantly affect convergence behavior and determine the suitability of different optimization methodologies.

4.1. Nonlinearity in electromagnetic response

Antenna performance metrics such as return loss, gain, radiation efficiency, bandwidth, and impedance are governed by electromagnetic field interactions that inherently exhibit nonlinear behavior. Small perturbations in antenna geometry, substrate properties, feed position, or material characteristics can lead to substantial changes in resonant frequency and radiation performance [65], [82], [85].

For example, the input impedance of an antenna can be expressed as (23).

$$Z_{in}(f, \mathbf{x}) = R(\mathbf{x}, f) + jX(\mathbf{x}, f), \tag{23}$$

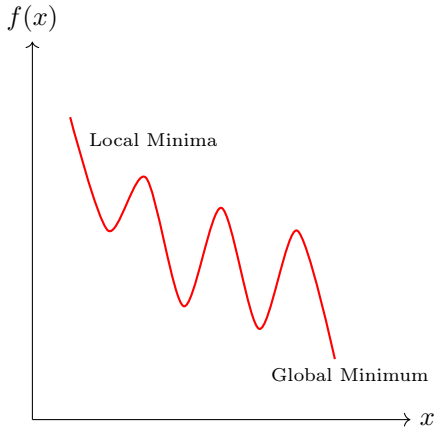


Figure 2: Conceptual multimodal optimization landscape.

In (24), $R(\mathbf{x}, f)$ and $X(\mathbf{x}, f)$ represent the frequency-dependent resistance and reactance, respectively.

The corresponding reflection coefficient is given by (24), where Z_0 denotes the characteristic impedance.

$$S_{11}(\mathbf{x}, f) = \frac{Z_{in}(\mathbf{x}, f) - Z_0}{Z_{in}(\mathbf{x}, f) + Z_0}, \tag{24}$$

Because both R and X vary nonlinearly with antenna geometry and operating frequency, the resulting optimization landscape contains irregular regions, steep gradients, plateaus, and discontinuities. These characteristics complicate gradient-based optimization and motivate the use of global search techniques [61], [83].

4.2. Multimodality and resonance effects

One of the defining characteristics of antenna optimization problems is multimodality. Multiple local optima arise naturally because antennas support several resonant modes corresponding to different current distributions and electromagnetic field patterns [58]–[60].

The objective function can be approximated as a superposition of resonant contributions (25):

$$f(\mathbf{x}) \approx \sum_{k=1}^{K} \alpha_k \phi_k(\mathbf{x}), \tag{25}$$

In (25),

- α_k represents the weighting coefficient,
- $\phi_k(\mathbf{x})$ denotes the contribution of the k-th resonant mode,
- K denotes the number of dominant resonances.

The interaction of multiple resonances produces highly complex objective surfaces characterized by numerous peaks and valleys. Consequently, optimization algorithms must balance exploration and exploitation to avoid premature convergence to local optima [56], [58], [63].

4.3. Local minima and optimization traps

The presence of multiple resonances creates numerous local minima throughout the search space. Conventional gradient-based methods frequently converge toward the

nearest stationary point and therefore exhibit strong dependence on initialization [56], [64]. Figure 2 shows a conceptual multimodal optimization landscape for illustrating multiple local minima and a global minimum.

Mathematically, a local minimum $\mathbf{x}^*$ satisfies (26).

$$\nabla f(\mathbf{x}^*) = 0, \qquad \nabla^2 f(\mathbf{x}^*) \succ 0, \tag{26}$$

In (26), $\nabla f(\mathbf{x}^*)$ denotes the gradient vector and $\nabla^2 f(\mathbf{x}^*)$ represents the Hessian matrix.

Although these conditions ensure local optimality, they do not guarantee that $\mathbf{x}^*$ corresponds to the global optimum. This limitation motivates the adoption of evolutionary algorithms, swarm intelligence methods, and hybrid optimization strategies capable of escaping local traps [58]–[60].

4.4. Parameter coupling and high dimensionality

Antenna design variables are often strongly coupled. Changes in one parameter simultaneously influence several performance metrics. For example, modifying patch length affects resonant frequency, impedance bandwidth, gain, and radiation efficiency.

Consequently, the objective function generally cannot be decomposed into independent subproblems (27):

$$f(\mathbf{x}) \neq \sum_{i=1}^{n} f_i(x_i), \tag{27}$$

Modern antenna systems such as massive MIMO arrays [78], [86], metasurfaces, phased arrays, and reconfigurable intelligent surfaces introduce hundreds or even thousands of optimization variables, giving rise to the well-known curse of dimensionality [61] [69], [73].

4.5. Noise and simulation uncertainty

Full-wave electromagnetic simulations introduce uncertainty due to mesh discretization, numerical approximations, solver tolerances, and finite computational precision [55], [84].

The observed objective function can therefore be represented as (28), where ϵ represents stochastic numerical error.

$$\tilde{f}(\mathbf{x}) = f(\mathbf{x}) + \epsilon, \tag{28}$$

Noise can distort gradient information, mislead optimization algorithms, and produce unstable convergence behavior. Robust optimization frameworks and surrogate-assisted approaches are therefore increasingly employed to mitigate uncertainty effects [55], [69].

4.6. Implications for optimization algorithm selection

The characteristics of antenna optimization landscapes directly influence algorithm selection:

- Multimodality → Global optimization methods such as GA, DE, PSO, GWO, and Cuckoo Search [56]–[59].
- Nonlinearity → Avoid reliance on purely gradient-based methods [62].
- High dimensionality → Surrogate-assisted optimization and AI-based dimensionality reduction [61], [69], [73].
- Noise and uncertainty → Robust and stochastic optimization frameworks [55].
- High computational cost → Space mapping and machine-learning-assisted optimization [69], [73].

Consequently, modern antenna optimization increasingly employs hybrid frameworks that combine evolutionary algorithms, swarm intelligence, surrogate models, and artificial intelligence to efficiently navigate complex search landscapes [61], [67]–[69], [73].

4.7. Summary of section

The optimization landscape in antenna design is fundamentally complex, characterized by strong nonlinearity, multimodality, high dimensionality, parameter coupling, simulation uncertainty, and substantial computational expense. These characteristics significantly influence algorithm performance and explain the growing adoption of advanced optimization methodologies including evolutionary computation, swarm intelligence, surrogate-assisted optimization, machine learning, and hybrid AI-driven frameworks. Understanding the structure of these landscapes provides the theoretical foundation for the taxonomy of optimization techniques presented in the following section.

5. Taxonomy of optimization techniques in antenna design

Optimization techniques used in antenna design can be classified according to their search mechanism, mathematical assumptions, computational cost, and suitability for electromagnetic simulation-driven problems. Since antenna responses are usually nonlinear, multimodal, and expensive to evaluate, no single optimization method is universally suitable for all antenna structures. Therefore, a structured taxonomy is necessary to identify the most appropriate algorithmic class for a given antenna design problem [10], [11]. Broadly, antenna optimization techniques can be categorized into deterministic methods, evolutionary algorithms, swarm intelligence methods, multi-objective optimization techniques, surrogate-assisted methods, space-mapping approaches, artificial intelligence-based techniques, robust optimization methods, and hybrid frameworks. Each category offers different strengths in terms of convergence speed, exploration capability, computational efficiency, and robustness [21], [50].

5.1. Deterministic optimization methods

Deterministic optimization methods rely on mathematical search directions derived from gradients, Hessian approximations, or local sensitivity information. Typical examples include gradient descent, Newton methods, quasi-Newton methods, sequential quadratic programming, and adjoint-based optimization [21], [22]. These methods are effective when the objective function is smooth and locally well behaved. In antenna design, deterministic methods are mainly used for local refinement after an approximate geometry has already been obtained. Their main advantage is fast convergence near a local optimum. However, because

Table 1: Optimization landscape characteristics and their implications for antenna optimization algorithms.

Landscape characteristics	Impact on optimization	Recommended approaches	References
Nonlinearity	Irregular objective surfaces and complex gradients	EA, PSO, Surrogate Models	[61, 82, 84]
Multimodality	Multiple local optima	GA, DE, PSO, GWO	[56, 58, 59, 60]
Local Minima	Premature convergence	Global Search Algorithms	[56, 62]
High Dimensionality	Curse of dimensionality	Surrogate Modeling, AI	[61, 69, 70]
Parameter Coupling	Non-separable objective functions	Hybrid Optimization	[61, 69]
Simulation Noise	Unstable convergence	Robust Optimization	[55, 84]
Computational Cost	Expensive EM evaluations	Space Mapping, Surrogates	[69, 73]

antenna optimization landscapes are usually multimodal, deterministic methods may converge to suboptimal local minima if the initial design is poorly selected.

5.2. Evolutionary algorithms

Evolutionary algorithms are population-based stochastic methods inspired by biological evolution. Genetic algorithms, differential evolution, and covariance matrix adaptation evolution strategy are commonly applied in antenna design [10], [14], [15]. These methods are particularly suitable for complex antenna structures because they do not require gradient information and can handle discrete, continuous, and mixed-variable design spaces. Evolutionary algorithms are widely used in array synthesis, fractal antenna design, slot geometry optimization, and multi-band antenna tuning. Their major limitation is the high number of electromagnetic simulations required, which can increase computational cost [28], [29].

5.3. Swarm intelligence methods

Swarm intelligence methods are inspired by collective behavior in natural systems. Particle swarm optimization, ant colony optimization, firefly algorithm, grey wolf optimizer, cuckoo search, and dragonfly algorithm belong to this category [16], [23]–[27]. These algorithms are attractive for antenna optimization because they are simple to implement and often converge faster than traditional evolutionary algorithms. Particle swarm optimization is especially popular in antenna array synthesis, beam steering, sidelobe level reduction, and impedance matching. However, swarm algorithms may suffer from premature convergence if population diversity is not properly maintained [57], [58].

5.4. Multi-objective optimization techniques

Many antenna design problems involve conflicting performance requirements [65], [66]. For example, antenna miniaturization may reduce bandwidth, while gain enhancement may increase antenna size or reduce beamwidth. Multi-objective optimization techniques are therefore essential for identifying trade-off solutions [17], [19], [20]. Methods such as NSGA-II, NSGA-III, SPEA2, and MOEA/D generate a set of non-dominated solutions known as the Pareto front. These solutions allow antenna designers to select the most suitable compromise among bandwidth, gain, radiation efficiency, physical size, sidelobe level, and mutual coupling [56], [60], [67].

5.5. Surrogate-assisted optimization

Surrogate-assisted optimization reduces computational cost by replacing expensive full-wave electromagnetic simulations with approximate predictive models. Common surrogate models include kriging, Gaussian process regression, radial basis functions, polynomial response surfaces, support vector regression, and neural networks [30]–[34], [51]. These methods are highly useful when each electromagnetic simulation requires significant time. Surrogate-assisted optimization is commonly used for broadband antennas, dielectric resonator antennas, metamaterial-inspired structures, and high-dimensional antenna arrays. The accuracy of the final design depends strongly on the quality of the surrogate model and the sampling strategy.

5.6. Space-mapping techniques

Space mapping is a specialized surrogate-based approach that establishes a relationship between a computationally inexpensive coarse model and an accurate fine electromagnetic model [33], [34]. The coarse model may be based on circuit approximation, transmission-line theory, or simplified electromagnetic simulation, while the fine model is usually a full-wave solver.

The main advantage of space mapping is that it reduces the number of expensive fine-model simulations while preserving design accuracy. This approach is particularly useful when a physically meaningful low-fidelity model is available.

5.7. AI-based optimization

Artificial intelligence-based optimization uses machine learning and deep learning models to predict antenna performance, accelerate design exploration, or generate antenna geometries from desired specifications [12], [18], [37]–[39], [42], [44]. AI-based methods include neural-network forward models, inverse design networks, generative models, reinforcement learning, and physics-informed neural networks.

These approaches are increasingly important for intelligent antenna design, reconfigurable antennas, metasurface synthesis, and real-time adaptive systems. However, their practical reliability depends on training data quality, physical consistency, and generalization to unseen antenna configurations.

5.8. Robust and uncertainty-aware optimization

Robust optimization considers variations caused by manufacturing tolerances, material uncertainty, temperature change, bending deformation, and environmental loading. Instead of optimizing only nominal performance, robust optimization seeks designs that maintain acceptable performance under uncertain conditions [53]–[55].

This category is especially important for industrial antenna design, wearable antennas, biomedical implants, and antennas fabricated using low-cost manufacturing processes. Methods such as Monte Carlo analysis, polynomial chaos expansion, stochastic collocation, and sensitivity analysis are often integrated with optimization algorithms.

5.9. Hybrid optimization frameworks

Hybrid optimization frameworks combine two or more optimization strategies to exploit their complementary strengths. A common approach is to use a global optimizer such as GA, PSO, or DE for broad exploration, followed by a deterministic method for local refinement. Another strategy combines surrogate modeling with evolutionary search and full-wave validation [11] [30], [31], [50].

Hybrid frameworks are particularly effective for antenna optimization because they balance exploration, exploitation, computational efficiency, and design accuracy. They are increasingly used in complex antenna systems where direct optimization is computationally prohibitive.

Table 2 summarizes the major categories of antenna optimization techniques and their typical applications.

5.10. Summary of section

The taxonomy shows that antenna optimization requires different algorithmic strategies depending on the design objective, computational budget, and physical complexity of the antenna structure. Deterministic methods are useful when local sensitivity information is available, while evolutionary and swarm-based methods are preferred for global search in multimodal landscapes. Multi-objective methods are required when conflicting performance metrics must be balanced, whereas surrogate-assisted and AI-based techniques are increasingly important for reducing electromagnetic simulation cost. Based on this classification, the following sections examine each major optimization family in greater technical detail, beginning with deterministic optimization methods.

6. Evolutionary algorithms in antenna optimization

Evolutionary algorithms are a class of population-based stochastic optimization methods inspired by the principles of natural selection and biological evolution. These algorithms are particularly well suited for antenna optimization problems due to their ability to handle nonlinear, multimodal, and non-differentiable objective functions without requiring gradient information [10], [14], [29].

Unlike deterministic methods, evolutionary algorithms explore the search space using a population of candidate solutions, enabling global exploration and reducing the likelihood of premature convergence to local minima. This property makes them highly effective for complex antenna design problems involving fractal geometries, reconfigurable antennas, antenna arrays, and metamaterial-based structures.

6.1. General Evolutionary Optimization Framework

A general evolutionary optimization process can be described as follows:

1. Initialize a population $\mathcal{P}_0 = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$
2. Evaluate fitness $f(\mathbf{x}_i)$ using EM simulation
3. Apply selection, crossover, and mutation operators
4. Generate new population $\mathcal{P}_{k+1}$
5. Repeat until convergence criteria are satisfied

The fitness function is typically derived from antenna performance metrics such as return loss, gain, bandwidth, or radiation efficiency.

6.2. Genetic Algorithms (GA)

Genetic algorithms are among the earliest and most widely used evolutionary techniques in antenna design [10], [14]. In GA, each candidate solution is encoded as a chromosome, which may represent antenna parameters such as dimensions, feed positions, or array excitation coefficients.

The evolution process consists of three primary operators:

- Selection: Individuals are selected based on fitness:

$$P_i = \frac{f_i}{\sum_{j=1}^{N} f_j} \tag{29}$$

- Crossover: Two parent solutions exchange information:

$$\mathbf{x}_{child} = \alpha \mathbf{x}_1 + (1-\alpha)\mathbf{x}_2 \tag{30}$$

- Mutation: Random perturbation is introduced:

$$x_i' = x_i + \delta \tag{31}$$

Genetic algorithms have been successfully applied in:

- Antenna array synthesis and thinning.
- Fractal antenna design.
- Multi-band and wideband antenna optimization.
- Yagi–Uda antenna parameter tuning.

Despite their robustness, GA methods often require a large number of function evaluations, making them computationally expensive when coupled with full-wave EM solvers.

6.3. Differential Evolution (DE)

Differential evolution is a powerful and efficient evolutionary algorithm for continuous optimization problems [15, 47]. It generates new candidate solutions by combining existing population members:

$$\mathbf{v}_i = \mathbf{x}_{r1} + F(\mathbf{x}_{r2} - \mathbf{x}_{r3}), \tag{32}$$

where F is a scaling factor and $r1, r2, r3$ are randomly selected indices. The trial vector $\mathbf{v}_i$ is then compared with the current solution:

$$\mathbf{x}_i^{new} = \begin{cases} \mathbf{v}_i, & \text{if } f(\mathbf{v}_i) < f(\mathbf{x}_i) \\ \mathbf{x}_i, & \text{otherwise} \end{cases} \tag{33}$$

Table 2: Taxonomy of optimization techniques used in antenna design.

Category	Representative methods	Main strengths	Typical antenna Applications
Deterministic optimization	Gradient descent, Newton, quasi-Newton, SQP, adjoint methods	Fast local convergence and efficient refinement	Fine tuning of antenna dimensions and impedance matching
Evolutionary algorithms	GA, DE, CMA-ES	Strong global exploration and gradient-free search	Fractal antennas, array synthesis, slot optimization
Swarm intelligence	PSO, GWO, firefly, cuckoo search, dragonfly algorithm	Simple implementation and effective population search	Beam steering, sidelobe reduction, array thinning
Multi-objective optimization	NSGA-II, NSGA-III, SPEA2, MOEA/D	Pareto-front generation for conflicting objectives	Bandwidth–gain–size trade-off analysis
Surrogate-assisted optimization	Kriging, RBF, Gaussian process, polynomial response surface, neural surrogates	Reduced full-wave simulation cost	Broadband and high-dimensional antenna design
Space mapping	Coarse-to-fine model mapping	Efficient use of low- and high-fidelity models	Microwave and antenna parameter optimization
AI-based optimization	Deep learning, inverse design, reinforcement learning, PINNs	Fast prediction and intelligent design automation	Reconfigurable antennas, metasurfaces, inverse synthesis
Robust optimization	Monte Carlo, polynomial chaos, stochastic optimization	Tolerance-aware and reliable design	Wearable, biomedical, and industrial antennas
Hybrid frameworks	GA–local search, PSO–surrogate, Bayesian–EM validation	Balanced exploration, exploitation, and efficiency	Complex simulation-driven antenna optimization

The DE is particularly effective for antenna optimization due to strong global search capability, simple implementation and robust convergence behavior.

It has been widely used in optimizing antenna dimensions, feed networks, and array configurations.

6.4. Covariance matrix adaptation evolution strategy

CMA-ES is an advanced evolutionary strategy that adapts the covariance matrix of the search distribution during optimization [28]. It samples candidate solutions from a multivariate normal distribution:

$$\mathbf{x}_k \sim \mathcal{N}(\mathbf{m}_k, \mathbf{C}_k), \tag{34}$$

where $\mathbf{m}_k$ is the mean vector and $\mathbf{C}_k$ is the covariance matrix. The covariance matrix is updated based on successful search directions:

$$\mathbf{C}_{k+1} = (1 - c_c)\mathbf{C}_k + c_c \mathbf{p}_c \mathbf{p}_c^T \tag{35}$$

CMA-ES is particularly suitable for high-dimensional antenna optimization, strongly coupled design variables and non-separable optimization problems.

Its ability to learn correlations between variables makes it highly effective for complex antenna geometries.

6.5. Convergence characteristics

Evolutionary algorithms generally exhibit stochastic convergence behavior. Unlike deterministic methods, they do not guarantee convergence to a global optimum but provide probabilistic convergence under certain assumptions [24].

The convergence rate depends on population size, mutation and crossover parameters, selection pressure and diversity preservation mechanisms. Premature convergence is a common issue, especially when population diversity is lost. Techniques such as adaptive mutation, elitism, and diversity control are used to mitigate this problem.

6.6. Applications in antenna design

Evolutionary algorithms have been extensively applied in various antenna engineering problems:

- Array synthesis: Optimization of element spacing and excitation coefficients for sidelobe suppression and beam shaping
- Fractal antennas: Automatic generation of self-similar geometries
- Reconfigurable antennas: Optimization of switching states and tunable elements
- MIMO systems: Minimization of mutual coupling and envelope correlation
- Metamaterial antennas: Optimization of periodic structures and unit cell parameters

These applications demonstrate the versatility of evolutionary algorithms in handling diverse antenna design challenges.

6.7. Advantages of evolutionary algorithms

- Global search capability
- No requirement for gradient information
- Ability to handle discrete and continuous variables
- Robustness in multimodal landscapes
- Flexibility in objective function formulation

6.8. Limitations

- High computational cost due to large number of EM simulations
- Slow convergence compared to deterministic methods
- Sensitivity to parameter tuning
- Lack of guaranteed optimality

6.9. Summary of section

Evolutionary algorithms play a critical role in antenna optimization due to their ability to explore complex and multimodal design spaces. While they are computationally expensive, their robustness and flexibility make them indispensable for global optimization tasks.

In practical antenna design workflows, evolutionary algorithms are often combined with surrogate models, local refinement techniques, or hybrid optimization frameworks to improve efficiency and convergence speed.

The next section discusses swarm intelligence techniques, which offer an alternative population-based optimization paradigm with faster convergence characteristics and simpler implementation.

7. Swarm intelligence techniques

Swarm intelligence techniques are population-based stochastic optimization methods inspired by the collective behavior of natural systems such as bird flocking, fish schooling, insect swarming, and predator hunting [16], [23]. These methods have gained significant attention in antenna optimization due to their simplicity, fast convergence, and ability to handle nonlinear and multimodal objective functions [24], [25]. Figure 3 illustrates swarm intelligence-based antenna optimization loop for optimized antenna design.

Unlike evolutionary algorithms that rely on genetic operators such as crossover and mutation, swarm intelligence methods update candidate solutions through cooperative information sharing among individuals in the population. This results in efficient exploration and exploitation of the search space, making SI techniques particularly suitable for electromagnetic optimization problems.

7.1. Particle swarm optimization

Particle swarm optimization is one of the most widely used swarm-based algorithms in antenna design [16], [24]. In PSO, each particle represents a candidate solution, and its movement in the search space is influenced by its own experience and the collective knowledge of the swarm. Algorithm 1 shows the procedure of antenna design by using PSO. The PSO has been extensively applied in antenna array synthesis and beamforming, sidelobe level reduction, impedance matching optimization and reconfigurable antenna tuning.

The velocity and position update equations are given by (36):

$$\mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + c_1 r_1(\mathbf{p}_i - \mathbf{x}_i^k) + c_2 r_2(\mathbf{g} - \mathbf{x}_i^k), \quad (36)$$

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1}, \quad (37)$$

where:

- w is the inertia weight,
- c_1, c_2 are cognitive and social coefficients,
- r_1, r_2 are random numbers in $[0, 1]$,
- $\mathbf{p}_i$ is the personal best position,
- $\mathbf{g}$ is the global best position.

Algorithm 1 PSO for antenna design.

```
Initialize particles x_i
while not converged do
    Evaluate fitness using EM simulation
    Update velocity v_i
    Update position x_i
    Update personal and global best
end while
Return optimal antenna design
```

7.2. Grey Wolf Optimizer (GWO)

The GWO is inspired by the leadership hierarchy and hunting behavior of grey wolves [26]. The population is divided into four categories: alpha (α), beta (β), delta (δ), and omega (ω). The GWO has demonstrated strong performance in antenna array thinning and pattern synthesis.

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_\alpha + \mathbf{X}_\beta + \mathbf{X}_\delta}{3} \quad (38)$$

7.3. Firefly Algorithm (FA)

The FA is inspired by the flashing behavior of fireflies [27]:

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \beta_0 e^{-\gamma r_{ij}^2}(\mathbf{x}_j^k - \mathbf{x}_i^k) + \alpha\epsilon_i \quad (39)$$

It is particularly effective for multimodal problems.

7.4. Other Swarm-Based Algorithms

Several other swarm intelligence algorithms have been explored in antenna optimization:

- Cuckoo Search (CS): Based on brood parasitism and Lévy flights [58]
- Dragonfly Algorithm (DA): Mimics static and dynamic swarming behaviors [57]
- Artificial Bee Colony (ABC): Based on foraging behavior of honey bees
- Ant Colony Optimization (ACO): Inspired by pheromone trail formation

These methods have been applied to antenna parameter tuning, array synthesis, and electromagnetic inverse problems.

7.5. Convergence behavior

Swarm intelligence algorithms exhibit stochastic convergence and rely on information sharing among individuals. Their convergence is influenced by:

- Balance between exploration and exploitation
- Parameter settings (e.g., inertia weight in PSO)
- Population diversity
- Problem dimensionality

Adaptive parameter tuning strategies are often used to improve convergence and prevent stagnation.

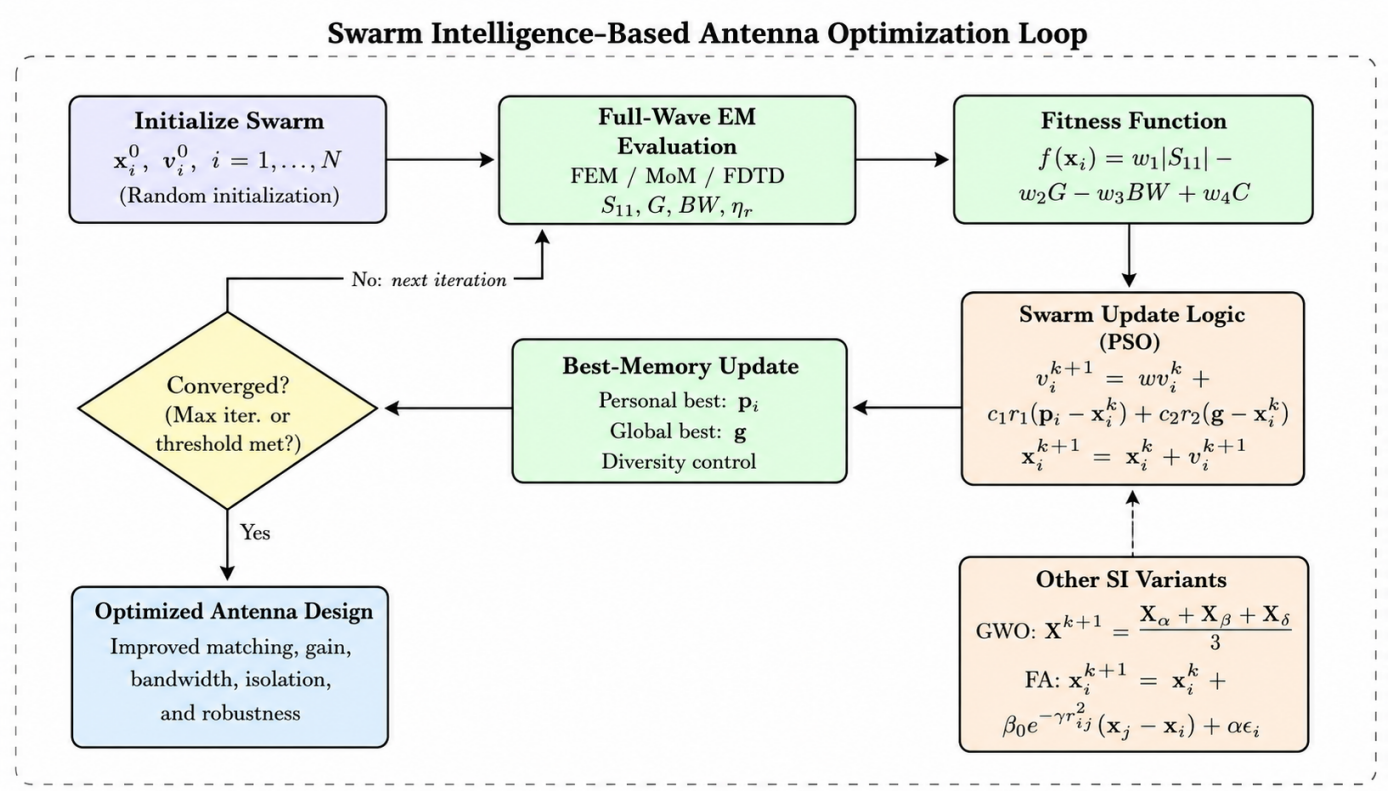


Figure 3: Swarm intelligence-based antenna optimization loop for optimized antenna design.

7.6. Applications in antenna design

Swarm intelligence methods have been successfully applied in:

- Array optimization: Beam steering, sidelobe suppression, null placement.
- Compact antenna design: Parameter tuning for miniaturized antennas.
- MIMO systems: Mutual coupling reduction and correlation control.
- Metasurface design: Optimization of unit cell parameters.
- Reconfigurable antennas: Switching state optimization.

These applications highlight the flexibility of SI techniques in handling diverse antenna design challenges.

7.7. Comparison with evolutionary algorithms

Swarm intelligence and evolutionary algorithms are two major classes of population-based stochastic optimization techniques widely applied in antenna optimization problems. Although both approaches aim to identify globally optimal solutions in nonlinear and multimodal search spaces, they differ significantly in their underlying search philosophy, information-sharing mechanisms, convergence behavior, and computational complexity.

Evolutionary algorithms emulate Darwinian evolution through operators such as selection, crossover, and mutation, whereas swarm intelligence methods mimic collective social behavior through cooperative information exchange among individuals in the population. Consequently, SI methods update candidate solutions using velocity-position dynamics or attraction-based interactions, while EAs rely on probabilistic genetic transformations to generate new populations.

In general, swarm intelligence algorithms exhibit faster convergence because information about promising regions is rapidly propagated throughout the swarm. Mathematically, PSO updates are governed by (40), whereas genetic algorithms rely on crossover (41) and mutation (42).

$$\mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + c_1r_1(\mathbf{p}_i - \mathbf{x}_i^k) + c_2r_2(\mathbf{g} - \mathbf{x}_i^k), \tag{40}$$

$$x_{\text{child}} = \alpha x_1 + (1 - \alpha)x_2, \tag{41}$$

$$x' = x + \delta. \tag{42}$$

From an antenna optimization perspective, the suitability of SI or EA depends strongly on the problem characteristics:

- PSO is highly effective for continuous antenna parameter tuning due to its fast convergence behavior.
- Genetic algorithms are preferred for topology optimization and discrete geometry synthesis because crossover and mutation naturally support structural diversity.
- Differential evolution offers a balance between convergence speed and robustness, making it suitable for broadband and array optimization problems.
- Swarm intelligence algorithms are computationally simpler and easier to implement for large-scale electromagnetic optimization.

The comparative characteristics of swarm intelligence and evolutionary algorithms are summarized in Table 3.

7.8. Advantages of swarm intelligence methods

- Fast convergence compared to evolutionary algorithms
- Simple implementation with fewer control parameters
- Effective for continuous optimization problems
- Good balance between exploration and exploitation

7.9. Limitations

- Premature convergence in complex multimodal landscapes.
- Sensitivity to parameter tuning.
- Reduced performance in very high-dimensional problems.
- Lack of theoretical convergence guarantees.

7.10. Summary of section

Swarm intelligence techniques provide an efficient alternative to evolutionary algorithms for antenna optimization, particularly when fast convergence is required. Their ability to exploit collective intelligence makes them well suited for continuous parameter tuning and array optimization problems.

8. Multi-objective optimization in antenna design

Modern antenna design inherently involves multiple conflicting performance objectives. As a result, antenna optimization problems are more accurately formulated as multi-objective optimization problems rather than single-objective ones [17]–[20].

8.1. Mathematical formulation of multi-objective problems

A general multi-objective antenna optimization problem can be expressed as (43):

$$\min_{\mathbf{x}\in\Omega} \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T, \tag{43}$$

In (43), The objectives are often conflicting, making it impossible to find a single solution that simultaneously optimizes all criteria AND each objective function represents a performance metric such as:

- $f_1(\mathbf{x})$: Return loss or impedance matching
- $f_2(\mathbf{x})$: Gain or directivity
- $f_3(\mathbf{x})$: Bandwidth
- $f_4(\mathbf{x})$: Radiation efficiency
- $f_5(\mathbf{x})$: Physical size or weight

8.2. Pareto Optimality Theory

In multi-objective optimization, solutions are evaluated based on Pareto dominance. A solution $\mathbf{x}_a$ is said to dominate another solution $\mathbf{x}_b$ if:

$$f_i(\mathbf{x}_a) \leq f_i(\mathbf{x}_b) \quad \forall i \tag{44}$$

and

$$f_j(\mathbf{x}_a) < f_j(\mathbf{x}_b) \quad \text{for at least one } j. \tag{45}$$

The set of non-dominated solutions forms the Pareto-optimal set, and its image in the objective space is called the Pareto front [19], [20]. Each point on the Pareto front represents a trade-off solution where improvement in one objective leads to degradation in at least one other objective. For a two-objective problem, the Pareto front can be visualized as a curve in the objective space. For example, consider gain and bandwidth trade-off (46):

$$\mathbf{F}(\mathbf{x}) = [\text{Bandwidth}(\mathbf{x}), -\text{Gain}(\mathbf{x})]. \tag{46}$$

The Pareto front typically exhibits a non-linear trade-off, indicating that significant improvement in gain may require substantial compromise in bandwidth.

8.3. Multi-objective evolutionary algorithms (MOEAs)

Multi-objective evolutionary algorithms are the most widely used methods for solving antenna optimization problems due to their ability to generate a diverse set of Pareto-optimal solutions in a single run.

8.3.1. NSGA-II (Non-dominated Sorting Genetic Algorithm II)

NSGA-II is one of the most popular MOEAs in antenna design [17]. It employs fast non-dominated sorting, rowding distance for diversity preservation and elitism to retain high-quality solutions.

The algorithm ranks solutions based on Pareto dominance and maintains diversity using crowding distance as expressed in (47).

$$d_i = \sum_{k=1}^{m} \frac{f_k^{i+1} - f_k^{i-1}}{f_k^{\max} - f_k^{\min}}. \tag{47}$$

NSGA-II has been extensively applied in:

- Broadband antenna design
- Multi-band antenna optimization
- MIMO antenna systems
- Array synthesis with sidelobe constraints

8.3.2. NSGA-III and MOEA/D

NSGA-III extends NSGA-II for many-objective problems by using reference points to maintain diversity [56]. MOEA/D decomposes a multi-objective problem into multiple scalar optimization subproblems is given by (48) [60], where λ is a weight vector and z^* is the ideal point. These methods are particularly useful for complex antenna design problems involving more than three objectives.

$$\min g(\mathbf{x}|\lambda) = \max_i \lambda_i |f_i(\mathbf{x}) - z_i^*|, \tag{48}$$

8.4. Trade-Off analysis in antenna design

Multi-objective optimization enables systematic exploration of trade-offs among antenna performance metrics:

- Gain vs Bandwidth: High gain antennas often exhibit narrow bandwidth
- Size vs Efficiency: Miniaturization reduces radiation efficiency

Table 3: Comparison between swarm intelligence and evolutionary algorithms in antenna optimization.

Criterion	Swarm intelligence	Evolutionary algorithms
Search mechanism	Collective social interaction and cooperative learning	Genetic evolution through crossover and mutation
Population update	Velocity-position or attraction-based updates	Selection, crossover, mutation operations
Convergence speed	Generally faster convergence	Moderate convergence speed
Exploration capability	Moderate exploration	Strong global exploration
Diversity preservation	Limited diversity control	Strong diversity through mutation
Risk of premature convergence	Higher for complex multimodal problems	Lower due to mutation diversity
Computational complexity	Lower implementation complexity	Higher computational complexity
Parameter sensitivity	Sensitive to inertia and acceleration coefficients	Sensitive to crossover and mutation rates
Best application scenario	Continuous parameter optimization	Discrete topology optimization
Typical antenna applications	Beamforming, tuning, impedance matching	Topology synthesis, fractal antennas, arrays
Representative algorithms	PSO, GWO, Firefly, ACO	GA, DE, CMA-ES

- Sidelobe Level vs Beamwidth: Lower sidelobes may widen the main beam
- Isolation vs Compactness (MIMO): Reducing mutual coupling increases spacing

These trade-offs are critical in applications such as:

- 5G and 6G antennas
- Satellite communication systems
- Wearable and implantable antennas
- Compact IoT devices

8.5. Objective aggregation techniques

In some cases, multiple objectives are combined into a single scalar objective using aggregation as (49), where w_i are weighting coefficients. However, this approach has limitations:

- Requires predefined weights
- Cannot capture non-convex Pareto fronts
- May bias toward certain objectives

$$f(\mathbf{x}) = \sum_{i=1}^{m} w_i f_i(\mathbf{x}), \tag{49}$$

8.6. Applications in antenna engineering

Multi-objective optimization has been widely applied in:

- Wideband antennas: Simultaneous optimization of bandwidth and return loss.
- MIMO systems: Trade-off between isolation, gain, and envelope correlation coefficient..
- Phased arrays: Beam steering, sidelobe suppression, and gain optimization.
- Metasurfaces: Control of reflection phase, amplitude, and polarization.

8.7. Advantages

- Provides multiple trade-off solutions
- Eliminates need for predefined weights
- Captures complex design relationships
- Enables informed engineering decisions

8.8. Limitations

- High computational cost due to population-based search
- Large number of solutions may require post-processing
- Difficult visualization for high-dimensional objectives

8.9. Summary of section

Multi-objective optimization has become an essential tool in antenna design due to the inherently conflicting nature of performance metrics. Pareto-based approaches provide a comprehensive understanding of trade-offs, enabling designers to select optimal solutions based on application-specific requirements. However, the high computational cost associated with multi-objective optimization motivates the integration of surrogate-assisted and AI-based techniques, which are discussed in the following sections.

9. Surrogate-assisted optimization and space mapping

One of the primary challenges in antenna optimization is the high computational cost associated with full-wave electromagnetic simulations. For complex antenna geometries, a single simulation using methods such as FEM, MoM, or FDTD may take from several minutes to hours. Surrogate-assisted optimization and space-mapping techniques address this limitation by reducing the number of expensive EM simulations while preserving solution accuracy [30]–[34]. Algorithm 2 refers a procedure for design antenna by using surrogate-assisted optimization.

9.1. Surrogate modeling framework

A surrogate model (also known as a metamodel or response surface) is an approximate representation of the true objective function is given by (50).

$$\hat{f}(\mathbf{x}) \approx f(\mathbf{x}), \tag{50}$$

In (50), $f(\mathbf{x})$ is evaluated using high-fidelity EM simulation and $\hat{f}(\mathbf{x})$ is a computationally inexpensive approximation.

The surrogate modeling process involves:

1. Sampling design points $\{\mathbf{x}_i\}$
2. Evaluating EM responses $f(\mathbf{x}_i)$
3. Constructing a predictive model $\hat{f}(\mathbf{x})$

4. Using $\hat{f}(\mathbf{x})$ for optimization

Algorithm 2 Surrogate-assisted antenna optimization

```
Generate initial design samples {x_i}
Evaluate EM responses f(x_i)
Train surrogate model f̂(x)
while not converged do
    Optimize surrogate model to obtain candidate x*
    Evaluate f(x*) using EM simulation
    Update surrogate model with new data
end while
Return optimized antenna design
```

The accuracy of the surrogate model depends on the sampling strategy, model selection, and dimensionality of the design space.

9.2. Kriging (Gaussian Process Regression)

Kriging is one of the most widely used surrogate modeling techniques in antenna optimization. It models the objective function as a Gaussian process as expressed by (51).

$$\hat{f}(\mathbf{x}) = \mu + Z(\mathbf{x}) \tag{51}$$

In (51), μ is the mean and $Z(\mathbf{x})$ is a Gaussian random process with covariance is given by (52).

$$\mathrm{Cov}(Z(\mathbf{x}_i), Z(\mathbf{x}_j)) = \sigma^2 \times \exp\left(-\sum_{k=1}^{n} \theta_k \left| x_{i,k} - x_{j,k} \right|^2 \right) \tag{52}$$

Kriging provides both prediction and uncertainty estimation, making it particularly suitable for adaptive sampling and Bayesian optimization [30], [31].

9.3. Radial Basis Function (RBF) Models

RBF models approximate the objective function using weighted radial kernels is given by (53), where $\phi(\cdot)$ is a radial basis function such as Gaussian, multiquadric, or inverse quadratic.

$$\hat{f}(\mathbf{x}) = \sum_{i=1}^{N} w_i \phi(||\mathbf{x} - \mathbf{x}_i||), \tag{53}$$

RBF models are computationally efficient and easy to implement, making them suitable for high-dimensional antenna optimization problems [32].

9.4. Polynomial response surface models

Polynomial models approximate the objective function using low-order polynomials (54):

$$\hat{f}(\mathbf{x}) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} x_i x_j. \tag{54}$$

These models are simple and computationally inexpensive but may lack accuracy for highly nonlinear antenna responses.

9.5. Neural network surrogates

Artificial neural networks provide flexible nonlinear mapping between input parameters and antenna performance:

$$\hat{f}(\mathbf{x}) = \sigma\left(\mathbf{W}_2 \cdot \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1) + \mathbf{b}_2\right), \tag{55}$$

Neural network surrogates are particularly useful for complex antenna geometries and large datasets [38], [42].

9.6. Bayesian optimization

Bayesian optimization has emerged as an effective strategy for antenna optimization when electromagnetic simulations are computationally expensive and the number of allowable function evaluations is limited [35], [36]. Unlike conventional population-based optimization methods that may require hundreds or thousands of simulations, Bayesian optimization seeks to identify promising antenna designs using a probabilistic surrogate model and an intelligent sampling strategy. Algorithm 3 shows the Bayesian optimization for antenna design. Bayesian optimization combines surrogate modeling with an acquisition function to guide sampling is given by (56). In (56), $\alpha(\mathbf{x})$ represents the acquisition function that balances exploration of uncertain regions and exploitation of promising solutions. Common acquisition strategies include Expected Improvement, Probability of Improvement, and Upper Confidence Bound methods.

$$\mathbf{x} * next = \arg\max * \mathbf{x} \alpha(\mathbf{x}), \tag{56}$$

A Gaussian Process model is commonly employed as the surrogate because it provides both a prediction of antenna performance and an estimate of prediction uncertainty. This uncertainty information enables the optimizer to efficiently determine whether additional simulations should focus on unexplored regions of the design space or refine already promising solutions.

Algorithm 3 Bayesian optimization for antenna design

```
Initialize dataset 𝒟
Train Gaussian process model
while budget not exhausted do
    Select next point x*next using acquisition function
    Evaluate f(x*next) via EM simulation
    Update dataset and retrain model
end while
Return best design
```

9.7. Adaptive sampling and model refinement

To improve surrogate accuracy, adaptive sampling strategies are used. These strategies select new sample points based on:

- Prediction uncertainty
- Expected improvement
- Model error

This iterative refinement process enhances model accuracy in critical regions of the design space.

9.8. Space mapping techniques

Space mapping is a powerful surrogate-assisted optimization approach that establishes a relationship between a coarse model and a fine EM model [33]. The coarse model is computationally inexpensive but less accurate, while the fine model is accurate but expensive. The optimization is performed using the coarse model, and corrections are applied using fine-model evaluations. Space mapping significantly reduces the number of expensive EM simulations while maintaining high accuracy.

The mapping is defined as (57), where $\mathbf{x}_c$ and $\mathbf{x}_f$ represent coarse and fine model parameters, respectively.

$$\mathbf{x}_c = P(\mathbf{x}_f), \tag{57}$$

9.9. Applications in antenna design

Surrogate-assisted optimization has been widely applied in:

- Broadband and ultra-wideband antenna design
- Dielectric resonator antennas
- Metamaterial and metasurface structures
- Antenna array optimization
- High-frequency millimeter-wave antenna systems

These methods enable efficient exploration of large design spaces with limited computational resources.

9.10. Advantages

- Significant reduction in computational cost.
- Efficient handling of expensive EM simulations.
- Capability to model complex nonlinear relationships.
- Compatibility with global optimization algorithms.

9.11. Limitations

- Accuracy depends on training data quality
- Curse of dimensionality for high-dimensional problems
- Model training overhead
- Risk of model mismatch in unexplored regions

9.12. Summary of section

Surrogate-assisted optimization and space mapping represent critical advancements in antenna design, enabling efficient optimization in computationally expensive environments. These methods bridge the gap between high-fidelity EM simulation and practical optimization requirements. In modern antenna engineering, surrogate models are often integrated with evolutionary algorithms, swarm intelligence techniques, and Bayesian optimization frameworks to achieve high efficiency and accuracy.

10. Artificial intelligence in antenna optimization

Artificial intelligence (AI) has emerged as a powerful paradigm for accelerating antenna optimization, particularly when conventional simulation-driven methods become computationally expensive. Unlike traditional optimization algorithms that repeatedly evaluate candidate designs using full-wave electromagnetic solvers, AI-based methods learn nonlinear relationships between antenna design variables and electromagnetic responses from data [12], [18], [37]–[39], [42], [44]. Once trained, these models can rapidly predict antenna performance, support inverse design, and enable adaptive tuning of reconfigurable antenna systems.

10.1. AI-based forward modeling

In forward modeling, a machine learning model approximates the mapping from antenna design parameters to electromagnetic responses [68]. This can be expressed as (58), in which $\mathbf{x}$ represents the antenna design vector and $\mathbf{y}$ represents performance outputs such as reflection coefficient, gain, bandwidth, radiation efficiency, or far-field pattern.A trained AI model approximates this relationship as (59).

$$\mathbf{y} = \mathcal{M}(\mathbf{x}), \tag{58}$$

$$\hat{\mathbf{y}} = \mathcal{M}_\theta(\mathbf{x}), \tag{59}$$

where θ denotes the trainable model parameters.

For neural-network-based modeling, the training objective is commonly formulated as (60).

$$\min_\theta \frac{1}{N} \sum_{i=1}^{N} \|\mathbf{y}_i - \mathcal{M}_\theta(\mathbf{x}_i)\|^2 . \tag{60}$$

After training, the model can replace repeated full-wave simulation during early-stage design exploration. This is particularly useful for broadband antennas, MIMO antennas, metasurfaces, dielectric resonator antennas, and millimeter-wave structures where EM simulations are computationally demanding.

10.2. DL for antenna prediction

Deep learning models extract hierarchical features is given by (61) and the applications are S_{11} prediction, radiation pattern estimation and gain and efficiency prediction [74], [75].

$$\mathbf{h}^{(l)} = \sigma(\mathbf{W}^{(l)}\mathbf{h}^{(l-1)} + \mathbf{b}^{(l)}) \tag{61}$$

Inverse design aims to determine antenna geometry from desired electromagnetic specifications. Instead of predicting performance from geometry, inverse design estimates the design vector that satisfies a target response is expressed by (62). This problem is more difficult than forward modeling because the inverse mapping is often non-unique. Multiple antenna geometries may produce similar return loss, gain, or radiation characteristics [76]. Therefore, inverse design is commonly formulated as an optimization problem (63).

$$\mathbf{x} = \mathcal{M}^{-1}(\mathbf{y}_{target}). \tag{62}$$

$$\min_{\mathbf{x}} \|\mathcal{M}_\theta(\mathbf{x}) - \mathbf{y}_{target}\|^2 + \lambda R(\mathbf{x}), \tag{63}$$

In (63), $R(\mathbf{x})$ is a regularization term that enforces physical feasibility, smoothness, manufacturability, or compactness.

Generative models, including variational autoencoders and generative adversarial networks, have also been explored for antenna topology synthesis. These models can generate candidate geometries that satisfy target specifications, but they must be constrained using electromagnetic validation to avoid physically invalid designs.

10.3. Physics-informed neural networks

Purely data-driven models may violate physical laws when evaluated outside the training domain. Physics-informed neural networks (PINNs) address this limitation by embedding governing physical constraints into the learning process [37]. In antenna optimization, this may involve incorporating Maxwell-based residuals, boundary conditions, or known resonance relationships into the loss function.

A general physics-informed loss function can be expressed as (64).

$$\mathcal{L} = \mathcal{L}_{data} + \lambda_p \mathcal{L}_{physics}, \tag{64}$$

$$\mathcal{L}_{data} = \frac{1}{N} \sum_{i=1}^{N} \|\mathbf{y}_i - \hat{\mathbf{y}}_i\|^2, \tag{65}$$

$\mathcal{L}_{physics}$ penalizes violation of electromagnetic constraints. For example, Maxwell's curl equations may be introduced as residual terms (66)-(67).

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{66}$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}. \tag{67}$$

By constraining the learning process with physics, PINNs can improve generalization, reduce training data requirements, and enhance trustworthiness in antenna design.

10.4. RL for adaptive and reconfigurable antennas

Reinforcement learning is suitable for adaptive and reconfigurable antenna systems where an agent interacts with an electromagnetic environment and learns optimal tuning actions [40], [41]. The antenna system can be modeled as a Markov decision process (68). Reinforcement learning for antenna tuning is shown in Algorithm 4.

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, R, \gamma), \tag{68}$$

In (68), $\mathcal{S}$ is the state space, $\mathcal{A}$ is the action space, P is the transition probability, R is the reward function, and γ is the discount factor.

For reconfigurable antenna tuning, the state may include operating frequency, channel condition, beam direction, or measured reflection coefficient. The action may represent switching states, phase-shifter values, varactor bias voltages, or beam-steering commands. A typical reward function may be defined as (69).

$$R_t = \alpha G_t - \beta |S_{11,t}| - \eta C_t, \tag{69}$$

In (69), G_t is gain, $|S_{11,t}|$ represents impedance mismatch, and C_t is a penalty for power consumption or switching cost.

RL-based antenna optimization is promising for:

- adaptive beam steering;
- frequency-reconfigurable antennas;
- intelligent reflecting surfaces;
- dynamic impedance matching;
- real-time wireless environment adaptation.

Algorithm 4 Reinforcement learning for antenna tuning

```
Initialize policy π
while not converged do
    Observe state s_t
    Select action a_t
    Apply antenna tuning
    Receive reward R_t
    Update policy π
end while
```

10.5. AI-driven optimization algorithm

Artificial intelligence-driven optimization frameworks have emerged as a powerful alternative to conventional optimization methods for antenna design. Algorithm 5 shows a common procedure for AI-driven antenna optimization framework. The general AI-driven optimization process consists of five stages: data generation, model training, performance prediction, candidate selection, and electromagnetic validation. Initially, a database of antenna designs is generated using full-wave electromagnetic simulations. The collected data are subsequently used to train a predictive model $\mathcal{M}_\theta$, where θ denotes the trainable parameters. During optimization, the trained model predicts antenna performance without requiring computationally expensive electromagnetic simulations. Promising solutions are then validated using high-fidelity EM solvers and incorporated into the training database through an active-learning loop.

Mathematically, the AI model approximates the electromagnetic response function is given by (70).

$$\hat{\mathbf{y}} = \mathcal{M}_\theta(\mathbf{x}), \tag{70}$$

In (70), $\mathbf{x}$ represents antenna design parameters and $\hat{\mathbf{y}}$ denotes the predicted performance metrics such as gain, bandwidth, efficiency, and reflection coefficient.

The optimization objective can therefore be expressed as (71), where the AI model replaces expensive electromagnetic evaluations during most optimization iterations.

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} f(\mathcal{M}_\theta(\mathbf{x})), \tag{71}$$

The integration of AI models within the optimization loop substantially reduces the number of expensive electromagnetic simulations required during the search process. Recent studies have demonstrated that deep neural networks, Gaussian process models, graph neural networks, transformer architectures, and physics-informed neural networks can achieve significant reductions in optimization time while maintaining high prediction accuracy. Consequently, AI-driven optimization is expected to play a central role in future antenna design methodologies for 6G communications, intelligent reflecting surfaces, massive MIMO systems [79], [80], and adaptive electromagnetic structures.

Algorithm 5 AI-driven antenna optimization framework

```
Generate initial antenna dataset using EM simulations
Extract antenna parameters and performance features
Train AI model M_θ
Initialize optimization population X
repeat
    Predict antenna performance using M_θ
    Rank candidate solutions according to objective function
    Select elite designs using acquisition strategy
    Validate selected candidates using full-wave EM simulation
    Compute prediction error
    Update training dataset
    Retrain or fine-tune AI model
until convergence criterion satisfied
Return optimized antenna design x*
```

10.6. Advantages of AI-based antenna optimization

AI-based optimization provides several advantages:

- rapid performance prediction after training;
- reduced dependence on repeated full-wave simulations;
- capability for inverse design and topology generation;
- suitability for adaptive and reconfigurable systems;
- potential integration with digital twin frameworks.

These advantages make AI-based techniques attractive for next-generation antenna systems requiring fast and autonomous design exploration.

10.7. Limitations and practical challenges

Despite its potential, AI-based antenna optimization faces several limitations:

- requirement of large and diverse training datasets;
- poor generalization outside the training domain;
- limited interpretability of deep models;
- risk of physically invalid inverse designs;
- dependence on high-quality EM simulation or measurement data;
- need for final full-wave and experimental validation.

Therefore, AI should be considered as a complementary design accelerator rather than a complete replacement for electromagnetic theory and full-wave simulation.

10.8. Summary of AI in antenna optimization

Artificial intelligence has significantly expanded the scope of antenna optimization by enabling fast prediction, inverse synthesis, adaptive tuning, and physics-guided learning. Deep learning models provide flexible nonlinear approximation, inverse networks support automated geometry generation, PINNs improve physical consistency, and reinforcement learning enables real-time antenna adaptation. However, reliable AI-driven antenna design requires careful dataset generation, physical constraint integration, uncertainty analysis, and validation using high-fidelity EM tools. Future research is expected to emphasize hybrid physics-AI frameworks that combine electromagnetic insight, data-driven learning, and optimization efficiency.

11. Hybrid optimization frameworks

Despite the significant advances achieved by deterministic optimization, evolutionary algorithms, swarm intelligence methods, surrogate-assisted approaches, and artificial intelligence techniques, no single optimization framework consistently outperforms all others across the broad spectrum of antenna design problems. Antenna optimization typically involves highly nonlinear electromagnetic interactions, multiple conflicting objectives, computationally expensive full-wave simulations, manufacturing constraints, and uncertain operating environments. Consequently, recent research has increasingly focused on hybrid optimization frameworks that integrate complementary optimization strategies to leverage their individual strengths while mitigating their limitations [42], [46], [49], [50], [56].

Hybrid optimization frameworks combine two or more optimization paradigms within a unified search process. The primary objective is to improve global exploration, accelerate convergence, reduce computational complexity, and enhance robustness against local minima. Such frameworks have become particularly important in the design of advanced antenna systems for 5G, 6G, massive MIMO, intelligent reflecting surfaces, satellite communications, and adaptive electromagnetic structures.

11.1. Motivation for hybrid optimization

The effectiveness of hybrid optimization arises from the complementary characteristics of different optimization techniques. For example, evolutionary algorithms exhibit strong exploration capabilities but often require large numbers of function evaluations. In contrast, deterministic methods converge rapidly but are susceptible to local optima. Similarly, surrogate models significantly reduce computational costs but may introduce approximation errors, while full-wave electromagnetic solvers provide accurate evaluations at high computational expense.

Mathematically, a hybrid optimization framework can be represented as (72).

$$\mathbf{x}^* = \mathcal{H}\left(\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_n\right), \tag{72}$$

In (72), $\mathcal{H}$ denotes the hybrid optimization process and $\mathcal{O}_i$ represents individual optimization modules.

11.2. Classification of hybrid frameworks

Hybrid optimization methods can be broadly categorized into four classes:

1. Deterministic-evolutionary hybrids.
2. Evolutionary-swarm hybrids.
3. Surrogate-assisted hybrids
4. AI-integrated hybrids.

11.3. Evolutionary–swarm hybridization

One of the most widely adopted approaches combines evolutionary operators with swarm intelligence mechanisms. For example, genetic algorithms may be employed to maintain population diversity, while particle swarm optimization performs local exploitation.

The hybrid update process may be expressed as (73),where α controls the contribution of each optimizer.

$$\mathbf{x}_{new} = \alpha \mathbf{x}_{GA} + (1-\alpha)\mathbf{x}_{PSO}, \tag{73}$$

11.4. Surrogate-assisted hybrid optimization

Surrogate-assisted optimization frameworks integrate computationally inexpensive predictive models with high-fidelity electromagnetic simulations. The optimization process alternates between surrogate evaluations and periodic electromagnetic validation is given by (74), where $\hat{f}$ denotes the surrogate model and f represents the full-wave EM response. Such frameworks can reduce computational cost by more than 80% for complex antenna optimization tasks.

$$\hat{f}(\mathbf{x}) \approx f(\mathbf{x}), \tag{74}$$

11.5. AI-integrated hybrid optimization

Recent studies increasingly integrate deep learning, reinforcement learning, and physics-informed neural networks into traditional optimization frameworks. AI models provide rapid prediction capabilities, while optimization algorithms perform global search and refinement.

The general optimization objective can be written as (75), where $\mathcal{L}_{AI}$ represents the AI-derived guidance term.

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} [f(\mathbf{x}) + \lambda \mathcal{L}_{AI}] \tag{75}$$

11.6. Hybrid optimization algorithm

Hybrid optimization frameworks combine the complementary strengths of evolutionary algorithms, swarm intelligence techniques, surrogate models, and artificial intelligence to achieve efficient antenna synthesis in complex electromagnetic design spaces.

Algorithm 6 shows a procedure of hybrid AI-surrogate-evolutionary antenna optimization. The objective is to balance global exploration, local exploitation, computational efficiency, and solution robustness while minimizing the number of expensive electromagnetic simulations.

Let

$$\mathbf{x}^* = \mathcal{H}(\mathcal{O}_{EA}, \mathcal{O}_{SI}, \mathcal{O}_{SM}, \mathcal{O}_{AI}), \tag{76}$$

In (76), $\mathcal{O}_{EA}$, $\mathcal{O}_{SI}$, $\mathcal{O}_{SM}$, and $\mathcal{O}_{AI}$ denote evolutionary optimization, swarm intelligence, surrogate modeling, and artificial intelligence modules, respectively.

The hybrid framework simultaneously exploits the global search capability of evolutionary and swarm-intelligence algorithms, the computational efficiency of surrogate models, and the predictive power of artificial intelligence. Compared with standalone optimization methods, hybrid approaches generally achieve faster convergence, improved solution quality, enhanced robustness, and significantly reduced electromagnetic simulation cost. Consequently, hybrid optimization has emerged as one of the most promising paradigms for future antenna design problems involving massive MIMO systems [77]-[81], intelligent reflecting surfaces, terahertz communications, metasurfaces, and AI-native 6G wireless networks.

Algorithm 6 Hybrid AI-surrogate-evolutionary antenna optimization

```
Generate initial antenna population X_0
Perform full-wave EM simulations
Construct initial training dataset D
Train surrogate model f̂(x)
Train AI predictor M_θ
Initialize iteration counter k = 0
repeat
    Global exploration using EA/SI operators
    Generate candidate antenna designs
    Predict performance using f̂(x) and M_θ
    Rank candidate solutions
    Select elite designs
    Perform local refinement using gradient/SQP search
    Validate elite solutions using full-wave EM simulation
    Compute prediction error
    Update dataset: D ← D ∪ D_new
    Retrain surrogate model
    Fine-tune AI model
    Update global best solution x*
    k ← k + 1
until Maximum iteration reached or |f^{k+1} − f^k| < ε
Return optimized antenna design x*
```

11.7. Advantages and challenges

Hybrid frameworks offer several advantages:

- Improved exploration–exploitation balance
- Faster convergence
- Reduced EM simulation cost
- Enhanced robustness
- Better scalability

However, challenges remain in algorithm coordination, parameter tuning, computational overhead, and interpretability.

11.8. Summary of section

Hybrid optimization represents the current state-of-the-art in antenna optimization research. The integration of optimization theory, surrogate modeling, and artificial intelligence is expected to become the dominant paradigm for future antenna systems operating in highly dynamic communication environments.

12. Comparative analysis and algorithm selection guidelines

The broad spectrum of optimization algorithms reviewed throughout this paper demonstrates that each technique possesses distinct strengths, limitations, and areas of applicability. Consequently, the selection of an appropriate optimization strategy depends strongly on the antenna design objectives, computational budget, dimensionality of the search space, electromagnetic complexity, and required solution accuracy [11], [16], [17].

Traditional deterministic optimization methods offer rapid convergence and strong local exploitation capabilities but often struggle in highly nonlinear electromagnetic landscapes. In contrast, evolutionary and swarm intelligence algorithms provide superior global search capability and robustness against local minima, making them particularly attractive for complex antenna synthesis problems. More recently, surrogate-assisted optimization and artificial intelligence techniques have emerged as promising alternatives capable of significantly reducing computational cost while maintaining optimization accuracy [25]–[27], [37], [45]. Table **??** summarizes the major characteristics of optimization algorithms commonly employed in antenna design.

12.1. Performance analysis

The comparative analysis reveals several important observations.

First, deterministic optimization methods remain highly effective for smooth and differentiable objective functions. Their fast convergence characteristics make them attractive for local refinement and constrained optimization problems. However, their reliance on gradient information limits their applicability to highly nonlinear antenna structures [16], [17].

Second, evolutionary algorithms exhibit excellent global search capability and robustness against local minima. Genetic algorithms and differential evolution remain among the most frequently adopted techniques for topology optimization and broadband antenna design [10] ,[14], [24], [42], [62].

Third, swarm intelligence methods provide an attractive balance between convergence speed and implementation simplicity. Particle swarm optimization continues to be one of the most widely used techniques in antenna optimization due to its efficient search dynamics and low computational complexity [18], [19], [40], [43], [44].

Fourth, multi-objective evolutionary algorithms such as NSGA-II and MOEA/D have become indispensable for modern antenna design, where multiple conflicting objectives must be optimized simultaneously [14], [15], [56], [57].

Fifth, surrogate-assisted optimization has emerged as a practical solution for computationally expensive electromagnetic problems. By replacing expensive full-wave simulations with predictive models, these approaches dramatically reduce optimization time while maintaining acceptable accuracy [25]– [29], [49], [50].

Finally, artificial intelligence techniques are rapidly transforming antenna optimization by enabling inverse design [71], real-time prediction, adaptive tuning, and autonomous optimization capabilities [32], [34], [39], [45], [61].

12.2. Emerging trends

Analysis of recent literature reveals a clear transition from conventional optimization methodologies toward intelligent and hybrid optimization frameworks [32], [37], [45], [61], [67]. Three dominant research trends can be identified:

1. AI-assisted optimization for inverse antenna design and rapid performance prediction [37], [39], [45], [61].
2. Physics-informed machine learning frameworks that integrate Maxwell's equations directly into the optimization process [32].
3. Hybrid optimization architectures that combine surrogate models, artificial intelligence, and global search algorithms to achieve improved computational efficiency and optimization robustness [26], [29], [45], [67].

These developments indicate a fundamental shift toward intelligent, adaptive, and self-learning optimization systems capable of addressing the increasing complexity of next-generation antenna technologies.

12.3. Summary of comparative analysis and algorithm selection guidelines

The comparative study demonstrates that no single optimization algorithm consistently outperforms all others across every antenna design scenario as shown in Table 4. Instead, the effectiveness of an optimization strategy depends on the dimensionality of the design space, electromagnetic complexity, computational resources, and optimization objectives. Deterministic methods remain valuable for local refinement, evolutionary algorithms provide strong global exploration, swarm intelligence techniques offer rapid convergence, surrogate models reduce computational cost, and artificial intelligence enables unprecedented levels of automation and adaptability. Although no universally optimal optimization algorithm exists, practical antenna design scenarios often favor specific techniques. Table 5 provides a decision-support framework for selecting optimization strategies according to application requirements.

13. Future research directions

The rapid evolution of wireless communication technologies, intelligent electromagnetic systems, and computational intelligence is fundamentally transforming antenna design methodologies. Future antenna optimization research is expected to move beyond conventional simulation-driven approaches toward autonomous, intelligent, and physics-aware optimization ecosystems capable of supporting next-generation communication infrastructures. Based on the analysis presented throughout this review, several promising research directions can be identified.

13.1. AI-Native antenna optimization

Artificial intelligence is expected to become a core component of future antenna design frameworks. Rather than serving as auxiliary prediction tools, future AI systems will increasingly participate directly in optimization, decision-making, and design generation processes. Deep generative models, transformer architectures, graph neural networks, and foundation models may enable automated antenna synthesis from high-level performance specifications.

Future optimization frameworks may be represented as (77), where $\mathcal{G}$ denotes a generative AI model capable of directly producing antenna geometries that satisfy desired performance objectives $\mathbf{y}_{target}$. Such inverse-design paradigms have the potential to dramatically reduce design cycles and enable real-time antenna development.

$$\mathbf{x}^* = \mathcal{G}(\mathbf{y}_{target}) \tag{77}$$

Table 4: Comparative analysis of optimization algorithms used in antenna design.

Technique	**Exploration**	**Exploitation**	**Complexity**	**Convergence**	**Typical antenna applications**
Gradient-based methods	Low	Very High	Low	Fast local convergence	Impedance matching, local refinement [16], [17]
SQP	Low	Very High	Moderate	Fast for smooth problems	Constrained antenna optimization [16]
GA	High	Moderate	High	Robust but slower	Topology synthesis, fractal antennas [10], [41]
DE	High	High	Moderate	Stable and reliable	Broadband and multiband antennas [42]
CMA-ES	Very High	High	High	Excellent for coupled variables	Large-scale antenna optimization [23]
PSO	Moderate	High	Low	Very fast	Array synthesis, beamforming [18], [19], [43]
GWO	High	Moderate	Moderate	Balanced convergence	Radiation pattern optimization [21]
FA	High	Moderate	Moderate	Good multimodal search	Nonlinear antenna optimization [22]
NSGA-II	High	Moderate	High	Efficient Pareto search	Gain-bandwidth-size trade-offs [14], [15]
MOEA/D	High	High	High	Scalable many-objective optimization	MIMO and metasurface optimization [57]
SO	Moderate	High	Low EM cost	Depends on surrogate accuracy	Computationally expensive EM problems [25], [26], [49], [50]
Space mapping	Moderate	High	Low	Rapid convergence	Microwave and antenna parameter optimization [28]
Bayesian Optimization	High	High	Moderate	Sample efficient	Expensive black-box optimization [13], [30], [31], [52]
DL-based optimization	High	High	High training cost	Fast inference	Inverse antenna design [33], [34], [37], [39], [45]
RL	Adaptive	Adaptive	High	Sequential optimization	Reconfigurable antennas [35, 36]
PINN	Moderate	High	Moderate	Physics-consistent	Electromagnetic inverse design [32], [72]
Hybrid optimization	Very High	Very High	High	Highly robust	Large-scale multi-objective optimization [11, 29, 45, 67]

Table 5: Algorithm selection guidelines for antenna optimization problems.

Optimization problem	**Recommended techniques**
Impedance matching	Gradient methods, SQP, PSO [16], [17], [18]
Broadband antenna optimization	DE, CMA-ES, Hybrid EA [23, 42]
Fractal antenna synthesis	GA, DE [5], [10], [42]
Array pattern synthesis	PSO, GWO, NSGA-II [19], [21], [15]
MIMO antenna optimization	NSGA-II, MOEA/D, Hybrid Optimization [14], [57], [11]
Metasurface design	Deep Learning, PINNs, Bayesian Optimization [32], [38], [45]
Computationally expensive EM optimization	Kriging, Space Mapping, Surrogate Models [25], [28], [49], [50]
Reconfigurable antennas	Reinforcement Learning, Hybrid AI [2], [35], [36]
6G intelligent antenna systems	AI-integrated Hybrid Frameworks [3], [45], [61], [67], [87]
Large-scale many-objective optimization	MOEA/D, NSGA-II, Hybrid Optimization [14], [57], [67]

13.2. Physics-informed AI

One of the most promising directions involves the integration of electromagnetic theory with machine learning models. Physics-informed neural networks and hybrid electromagnetic-AI frameworks provide a pathway toward improving generalization, reliability, and interpretability.

Future models are expected to simultaneously minimize prediction and physics residual errors (78):

$$\mathcal{L} = \mathcal{L}_{data} + \lambda \mathcal{L}_{physics}, \tag{78}$$

In (78), $\mathcal{L}_{physics}$ enforces compliance with Maxwell's equations and electromagnetic boundary conditions.

Such approaches are expected to overcome many limitations of purely data-driven optimization frameworks.

13.3. Digital Twin-Based Antenna Optimization

Digital twin technology is emerging as a transformative concept for intelligent engineering systems. Future antenna optimization environments may employ real-time digital replicas capable of continuously updating design parameters based on operational feedback.

The integration of digital twins with optimization algorithms may enable:

- Real-time performance monitoring.
- Adaptive parameter tuning.
- Predictive maintenance.
- Autonomous optimization under changing environments.

Digital twin frameworks are expected to become particularly important for satellite communication systems, adaptive radar platforms, and intelligent wireless infrastructures.

13.4. Optimization for 6G and beyond

Future wireless systems will impose unprecedented requirements on antenna performance. Emerging 6G technologies will require simultaneous support for:

- Terahertz communications.
- Extremely large MIMO systems.
- Reconfigurable intelligent surfaces.
- Integrated sensing and communication.
- AI-native network architectures.
- Space-air-ground integrated networks.

These applications will significantly increase optimization complexity, necessitating highly scalable and intelligent optimization frameworks capable of handling large-dimensional design spaces and dynamic operating conditions.

13.5. Autonomous self-optimizing antennas

Current optimization approaches are predominantly offline. Future systems are expected to operate autonomously and continuously adapt to environmental changes.

Reinforcement learning and adaptive control techniques may enable self-optimizing antennas characterized by:

- Dynamic beam steering.
- Real-time impedance tuning.
- Autonomous spectrum adaptation.
- Environmental awareness.
- Self-healing capabilities.

Such systems represent an important step toward fully cognitive radio and intelligent communication environments.

13.6. Quantum-inspired and quantum optimization

Quantum computing and quantum-inspired optimization algorithms represent emerging research directions with potentially transformative implications for antenna design.

Quantum optimization approaches may offer advantages in:

- Large-scale combinatorial optimization.
- Electromagnetic inverse problems.
- Many-objective optimization.
- Global search in complex design spaces.

Although practical implementation remains limited, quantum-enhanced optimization may become increasingly relevant as quantum hardware matures.

13.7. Explainable and trustworthy optimization

As optimization frameworks become increasingly AI-driven, concerns regarding transparency, reliability, and trustworthiness become more important.

Future research should focus on:

- Explainable artificial intelligence (XAI).
- Uncertainty-aware optimization.
- Interpretable surrogate models.
- Trustworthy autonomous design systems.

13.8. Sustainable and green optimization

Energy efficiency and environmental sustainability are emerging priorities in communication engineering. Future optimization frameworks should explicitly consider energy consumption, material usage, manufacturing cost, and lifecycle sustainability.

Optimization objectives may therefore include (79):

$$f(\mathbf{x}) = \alpha E + \beta C + \gamma M, \tag{79}$$

where E, C, and M denote energy consumption, economic cost, and material utilization, respectively.

Such formulations align with broader sustainability goals for future wireless infrastructures.

13.9. Summary of future directions

The future of antenna optimization lies in the convergence of artificial intelligence, physics-informed modeling, digital twins, quantum computing, and autonomous optimization systems. Hybrid intelligent frameworks capable of integrating electromagnetic knowledge, real-time adaptation, and computational intelligence are expected to define the next generation of antenna design methodologies.

14. Conclusion

Antenna optimization has evolved from classical deterministic tuning techniques to sophisticated computational intelligence frameworks capable of addressing highly nonlinear, multimodal, and computationally intensive electromagnetic design problems. This review has presented a comprehensive examination of the theoretical foundations, historical evolution, mathematical formulations, optimization landscapes, and contemporary optimization methodologies employed in antenna design.

The study first reviewed the historical development of antenna optimization and established the mathematical foundations underlying electromagnetic optimization problems. The complex characteristics of antenna optimization landscapes, including multimodality, strong parameter coupling, high dimensionality, and conflicting objectives, were then analyzed to motivate the need for advanced optimization strategies.

A comprehensive taxonomy of optimization techniques was subsequently presented, covering deterministic optimization methods, evolutionary algorithms, swarm intelligence techniques, multi-objective optimization frameworks, surrogate-assisted optimization approaches, and artificial intelligence-driven methodologies. Comparative analyses demonstrated that deterministic methods provide efficient local refinement, evolutionary algorithms offer strong global exploration capabilities, swarm intelligence methods achieve rapid convergence with low implementation complexity, and surrogate-assisted approaches substantially reduce computational costs associated with full-wave electromagnetic simulations.

The review further highlighted the growing importance of artificial intelligence, physics-informed machine learning, reinforcement learning, and hybrid optimization frameworks. These emerging methodologies are increasingly capable of performing inverse design, autonomous optimization, adaptive tuning, and intelligent decision-making while maintaining computational efficiency. Hybrid optimization architectures that combine evolutionary algorithms, swarm intelligence, surrogate modeling, and artificial intelligence were identified as particularly promising due to their ability to balance exploration, exploitation, accuracy, and computational cost.

Several research gaps were also identified, including challenges related to scalability, computational complexity, AI generalization, explainability, fabrication-aware optimization, many-objective optimization, and real-time adaptive antenna design. Addressing these limitations will be critical for supporting future communication systems involving 5G, 6G, massive MIMO, intelligent reflecting surfaces, terahertz communications, and autonomous wireless infrastructures.

Overall, the analysis presented in this review indicates that antenna optimization is transitioning toward intelligent, physics-aware, data-driven, and autonomous design ecosystems. Future antenna engineering is expected to increasingly rely on hybrid computational frameworks that integrate electromagnetic theory, artificial intelligence, surrogate modeling, digital twins, and real-time optimization capabilities. Such developments will play a pivotal role in enabling the next generation of adaptive, efficient, and intelligent communication systems.

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CRediT authorship contribution statement

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